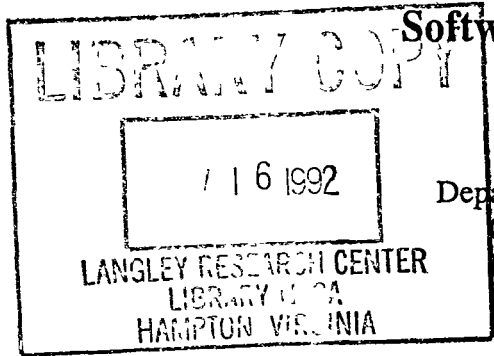


NASA-CR-187,688



NF00797

**Software Reliability Report**

3 1176 01355 7286

NAG 1-750

Larry Wilson

Department of Computer Science
Old Dominion University
Norfolk, VA 23529-0162

NASA-CR-187688
19920012098

There are many software reliability models which try to predict future performance of software based on data generated by the debugging process. Unfortunately, the models appear to be unable to account for the random nature of the data [8]. If we debug the same code multiple times and use one of the models to make predictions, we observe intolerable variance in the resulting reliability predictions. We believe that data replication can remove this variance in laboratory type situations and that it is less than scientific to talk about validating a software reliability model without considering replication. We also believe that data replication may prove to be cost effective in the real world, thus our research centers on verification of the need for replication and on methodologies for generating replicated data in a cost effective manner.

We have been pursuing the above in the context of the debugging graph [8] by simulation and experimentation. Simulation has been done for the Basic model and the Log-Poisson model [4] in a manner similar to that previously done for the Jelinski-Moranda and Moranda Geometric models [9]. That is reasonable values of the parameters have been assigned and used to generate simulated data which is then processed by the models in order to determine limitations on their accuracy. The experimentation is a continuation of that started by White and Harbison. These experiments exploit the existing software and program specimens which are in AIR-LAB to measure the performance of reliability models.

Keith Lirette conducted the simulation study of the basic model and has written a paper for his Master's project under our supervision. His conclusions were that the model performed well on all but the smallest sets of data, that data replication improved the accuracy of the model and that the confidence intervals provided by the model were inaccurate. We have enclosed his paper, as well as his data as part of this report but would like to add that he is overly generous in his assessment of the models perfor-

mance without replication. His work on the Log-Poisson model has not yet been written up.

Chris Cowles has conducted an empirical study measuring the performance of the Jelinski-Moranda Model and the Moranda Geometric Model using data from one of the LIC programs from RTI. He collected replicated data along the most probable (approximately) path through the debugging graph and fed this data into the two models. His conclusion was that the Geometric Model performed well when given accurate replicated data. This work used an environment at AIRLAB developed by Richard White and is based on code developed by Bernice Becher of NASA. His MS project paper and data is enclosed.

We plan to continue this investigation of the potential value of replication in the context of the debugging graph. The work so far has shown some value in this approach but we have not yet completed our work using the LIC programs, in that we have been working with a limited part of the debugging graph for one version of the LIC code. The next stage is to complete the error graph for this version prior to investigating the other useful version of the LIC software available. It is also our hope to apply similar techniques to the GCS specimens.

References

1. Z. Jelinski and P. Moranda, "Software Reliability Research," in Statistical Computer Performance Evaluation, W. Freiberger, Ed. New York: Academic, 1972, pp. 465-484.
2. Douglas R Miller, "Exponential Order Statistic Models of Software Reliability Growth", in IEEE Transactions on Software Engineering, vol SE-12, No 1, Jan 1986, pp 12-24.
3. P. B. Moranda, " Prediction of Software Reliability During Debugging," Proceedings of the 1975 Annual Reliability and Maintainability Symposium.
4. J. D. Musa A. Iannino and K. Okumoto, "Software Reliability: Measurement, Prediction, Application", McGraw-Hill, 1987.
5. C. V. Ramamoorthy and Farokh B. Bastani, "Software Reliability--- Status and Perspectives," in IEEE Transactions on Software Engineering, vol. SE-8No. 4, July 1982, pp 354-371.
6. G. J. Schick and R. W. Wolverton, "Achieving Reliability in Large Scale Software Systems," in Proc. 1974 Rel. and Maintainability Symp , Los Angeles, CA, Jan 1974, pp 302-319.
7. M. L. Shooman, "Probabilistic Models for Software Reliability Prediction," in Statistical Computer Performance Evaluation, W. Freiberger, Ed. New York: Academic, 1972, pp. 485-502
8. Wilson, Larry W. and Shen, Wenhui, "Software Reliability Perspectives", Old Dominion University Computer Science Department # TR-87-035, 1987.
9. Shen, Wenhui and Wilson, Larry W., "Simulation Studies of Software Reliability Models ", NASA Contractor Report 181889. Also released as ODU CS TR-89-10

MEASURING SOFTWARE RELIABILITY MODELS

Christopher Cowles
Old Dominion University Master's project
November, 1991

ABSTRACT

Accurate reliability models which predict future failure times of software are useful during software development. The accuracy of models is tested by comparing predictions which use the MTTFs of i errors with data from a known $i+1$ st error. Evaluations of a model's accuracy are found by tracing a path through the error graph of an existing piece of software. It is noted that a model's performance may be distorted when an abnormally large or small error is used in the sequence of errors, and that better results may be obtained by starting the model's run after the large dominant errors have been removed. It is also noted that to increase the accuracy of predictions, a sufficient number of inputs must be used to calculate failure rates and MTTFs.

INTRODUCTION

In any large software project, it is highly desirable to be able to predict the reliability of the software during the debugging phase. Only when the software is free (or nearly free to within a certain degree of tolerance) of errors is it ready to be released. But it is difficult to say exactly when a piece of software is nearly free of errors. Latent errors that remain may occur later when certain new demands are made on the software under untested (but valid) conditions. Reliability predictions themselves are at best good estimators of parameters such as the

number of initial errors or the expected time lapse until the next error.

Mathematical models of software reliability have been proposed in the past.[2][3][4] These may be used to estimate and/or predict software reliability. If a piece of software is reliable enough to allow, say, one error in a million runs to pass through to the output, it may be deemed acceptable, ie., good enough for release. The degree of acceptable reliability will vary greatly between software packages: for example, one error per year in a database application may be acceptable, but not for a real-time aviation application.

The Jelinski / Moranda and the Moranda Geometric De-Eutrophication models were used in this project. The JM model assumes that the errors found in the software are all the same size and have an equal probability of occurring. Further, it assumes that there are a finite number (N) of bugs to begin with, and that bug fixes are perfect, ie., no new errors are introduced when others are corrected. Given a sequence of n time intervals between $n+1$ successive errors, an estimate can be obtained for N by using maximum likelihood techniques.

The Moranda Geometric De-Eutrophication model assumes that the detection rate will geometrically decrease as each new error is found and removed. Estimates can be found for D (the initial failure rate) and for k (the rate at which the failure rate

declines when errors are fixed). These values can be used to calculate an estimate of the failure rate λ . Like the JM model it assumes perfect fixes, uses maximum likelihood techniques and uses the sequence of n inter-failure times as input. However, unlike the JM model, the Geometric De-Eutrophication assumes an infinite number of bugs.

This project uses an existing piece of software that has twelve known errors, corrections available for each error, and an oracle constructed from a trusted (gold) version of the software. Since errors can be arbitrarily removed from the original software, their removal can be done in any desired order. Inputs can be run on each of the versions, and each result can be judged by the oracle as success or failure. After recording the results of testing for each software version, a reliability rate, failure rate, and MTTF can be found. If all of the known errors are removed one at a time (in some order), a complete list of MTTFs can be obtained for this simulated debugging process. Once completed, a sub-sequence of the MTTFs can then be used as input to the two models in this project, and predicted MTTFs can be obtained. Since the actual MTTF will already have been calculated, the models' predicted MTTFs can be compared to the real MTTF. Thus, the accuracy of these two models can be studied.

Because the bugs can be removed selectively from this candidate software, comparing an estimated MTTF with an actual

MTTF is possible. These steps are not possible with software in the development process (normally no oracle exists at that time), but it is the performance of the software reliability models that is being evaluated here.

DESCRIPTION OF THE SOFTWARE

The software used in this project is called the LIC (Launch Interceptor Conditions) program. It is a Fortran program which simulates the testing of certain conditions, and the results of these tests are used to decide whether or not to issue a command to launch an interceptor missile. The multiple versions of the software, the gold version, the control program, are all written in VAX FORTRAN, were produced by the Research Triangle Institute [1], and exist at the AIRLAB facility at NASA Langley Research Center in Hampton, Virginia. The data collection processes were run on DEC VAXs using the VMS operating system.

The existing LIC software was chosen for this project for two reasons. First, work on software's error graph (described below) had already begun as part of a Master's project by White & Harbison[5] in 1990. This project involved work on the top and bottom levels of the error graph and further investigation was recommended. Second, most of the support apparatus needed for the experiment (gold program, control program, random number generator, etc.) already existed at NASA Langley.

Further, there were reasons for originally using the LIC program in the White / Harbison project. Not only had the errors that existed in the program been identified, but the corrections for these bugs had already been written. The versions of the software used in the White / Harbison work had the original errors left in place but commented out, and the inserted corrections were all identified with comment-tags. Thus newly required versions of the software used in this project (ie., versions with correction combinations not previously required) could easily be constructed.

The naming convention for the different software versions used in this project is the same as that used by White / Harbison. The individual bugs are identified by numbers 1 through 12. The number of the errors corrected ("fixed") in a version are identified in the name of the Fortran program; eg., Fix1235_AND_6 has five different bugs removed (namely bugs 1, 2, 3, 5, and 6), and Fix1_TO_9 has the first nine bugs removed. Examples of programs using this naming scheme appear in the appendices in this paper. The original oracle is based on a trusted program called Gold.

PREVIOUS WORK ON THE LIC PARTIAL ERROR GRAPH

A complete error graph is a graph with a single top node, a single bottom node, and some intermediate levels between them. The top node represents the software in its initial state with N

unknown bugs. The bottom node represents the final version of the software with all errors removed. Each horizontal level (with multiple nodes) between these single nodes represents a stage with some number of bugs corrected: the first level below the top node contains all software versions with one error corrected, the second level with all combinations of two errors corrected, etc. Between the nodes at each level an edge can be drawn to each node on the next level that represents a single step in the software debugging process. Hence, if there are N errors, level one will contain N nodes, each representing a single fix. From each node in level one there will be an edge to each of the $N-1$ nodes on the second level. Any walk from the top node to the bottom node constitutes a path through the error graph and represents a possible debugging sequence.

Each node can be labeled with its reliability measure, which is the ratio of successful runs to total tests for the version represented by that node. All inputs are randomly generated according to the input distribution and a successful run is defined as one where the results of that version of the software is the same as that of the gold version. A "delta reliability" number can be computed that indicates the percentage of change in reliability from the previous node. This delta reliability indicates the size of a single error within that previous version of the software.

A partial error graph is suitable to display information

about n known fixes. Since $N > n$, the partial error graph will be smaller and show only paths involving the known fixes. In the White / Harbison project, the levels with one bug corrected and $n-1$ bugs corrected were completed. For the one-bug level, the initial version of the software was separately fitted with each single correction, and the reliability of each such version was tested against the gold version. At the $(n-1)$ st level, each version was created by installing all but one fix. Again, the reliability of each of these versions was tested against the gold version.

DESCRIPTION OF THE SOFTWARE TESTING PROCESS

In order to obtain a sequence of MTTFs with which to judge the two models, it was decided to follow a single path through the error graph. This path was determined by identifying the largest error remaining at each level and correcting it to arrive at the node on the next level. This process was repeated until all of the known bugs had been removed from the LIC software. The determination of the size of the twelve errors was made from the first level of the White / Harbison paper. The ordering of bugs 1 through 12 by decreasing failure rate is: # 1, # 2, # 3, # 5, # 6, # 8, # 7, # 4, # 9, # 10, # 11, # 12.

It was assumed at each stage of this project that the next largest error still to be found along the single path would be the same as that in the ordered listing of bugs at level one.

This decision does not address the question of whether or not the removal of bugs one at a time would in any way affect the relative size of the remaining bugs. In order to determine the largest error of all of those errors left at each stage, it would have been necessary at each stage to couple each of the remaining errors separately with the corrections done so far along the path. Only by trying each possibility could one be sure that the identity of the largest error had been found. In this project, sixty-six ($11 + 10 + \dots + 1$) different software versions would have been required. Since each data point required one million random inputs and approximately three days of machine time, the time required to test along these lines made it prohibitive. It is suggested that there may be a need for further study.

Initially, one million random inputs were done on each software version being tested on the VAX and comparisons were made against the gold version. The results were categorized by the control program as success, abort, or failure; since both aborts and failures are disagreements with the results of the oracle, they are summed as "Total Failures" in Appendix A. For each group of runs, a corresponding second chart in the appendix shows the calculated reliability (number of successes divided by number of runs), failure rate (one minus reliability), and MTTF (the inverse of the failure rate).

Both reliability models were then checked using the empirical data. Appendix B lists the results of the JM model

using the data from one million runs. Estimates were derived from every possible sequence of two or more consecutive interfailure times which began with the MTTF for error #1. Also used were smaller sequences of consecutive fixes in the amounts of 3, 5 and 7 fixes.

In the JM model the actual value of N must, by definition, be larger than n , the number of errors used to predict N , since each of the n known errors is one of the N original errors. If, for example, the first six inter-time failures are used to find N , then the prediction of N includes these six errors. A predicted value of $N = 20$ in this case would indicate that 14 ($20 - 6$) errors remain. Also, when using any sub-sequence that does not include some of the initial errors, those errors must be excluded in determining the number of errors that remain. For example, suppose that when using the sub-sequence of MTTFs #5, #6 and #7 that N is predicted to be ten. This indicates that the total error content is ten beginning at error #5, ie., error #5 is considered to be the first error, since it was the first one used in this particular sub-sequence. Previous errors are not taken into account here. Since three MTTFs were used to make this prediction, seven errors ($10 - 3$) now remain.

The results of a typical removal of bugs in the JM model is shown in Figure 1 (next page). The graph approaches the x-axis in a step-wise manner; correcting each bug brings the otherwise horizontal graph closer by a step toward that axis. It is

possible for one of two things to happen: If the step-decreases were small enough each time, then the graph would not cross the axis on the next step and the prediction would be that the software still contains some number of errors. Otherwise, if the step sizes were large enough so that the next step would force the graph to be below the axis, then one would surmise that the software was now perfect (ie., free of errors).

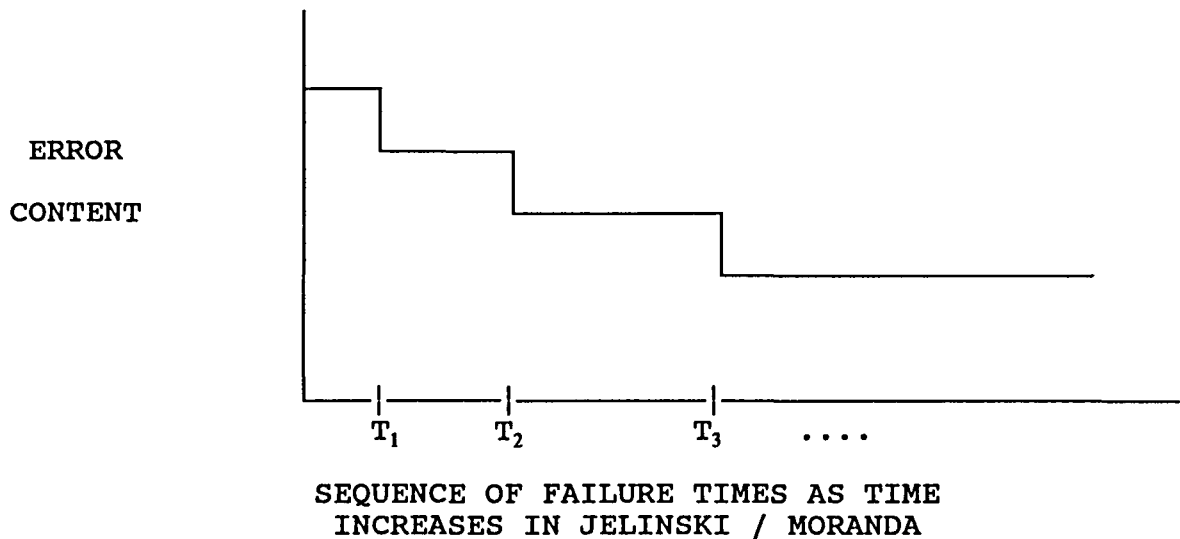


FIGURE 1.

This last case is a common occurrence with the JM predictions of N (the number of initial errors in the software) in this project. In Appendices B and D listing the results of the JM model, the software generally appears to be perfect. (Appendix D is the results without the large error #1. More is said below about the influence of the size of this error.) In Appendix B, because of the large size and subsequent impact of

correcting error #1, the next correction would be expected to produce an improvement on the same scale. It then appears as if the software has improved so much as to now be free of errors.

The data from the one million random inputs was next used on the Moranda Geometric De-Eutrophication model. It was decided to also run random inputs in the amounts of one, ten, one hundred, one thousand, ten thousand and one hundred thousand in order to be able to check the robustness of this model on data with varying quality. As a hypotheses, it was thought that more and more accurate predictions would be produced as the number of runs increased. All of these results are listed in Appendix C. As in the JM model, predictions are also made without using the influential error #1. These are listed in Appendix E. Finally, also included is Appendix F, listing the two Pascal programs used to run the reliability models.

RESULTS OF THE MODELS

The candidate software used in this project contained an extremely large and influential first error. The last four or five errors could all be classified as very small, with little effect on the predictability of the two models. In fact, even at runs of 100,000, the last four errors do not appear. Aside from these errors, the other corrections seem to improve the software in a steady, uniform manner.

Because of the influence of the first error, computations by each of the two models were repeated by first correcting error #1 and then using the resulting software version as the initial version. Hence data and consequently the models' predictions were calculated as if this large bug did not exist. In the JM model, checked again with the data from one million inputs, only the first X consecutive MTTF numbers (from 2 to 11 in this case, not 2 to 12 when error #1 was included) were used. Attempting the sequences of 3, 5, and 7 again would only shift the answers already obtained and listed in Appendix B. (For example, the JM result with error #1 on error sequence 4, 5, and 6 would be the same as the JM result without error #1 on error sequence 3, 4, and 5.) These new results are listed in Appendix D. The data from excluding error #1 and using inputs of one, ten, one hundred, etc. runs was also checked in the Geometric De-Eutrophication model. These new results are listed in Appendix E. The discussion of the results of all of the runs follows.

THE JELINSKI / MORANDA MODEL

The JM model predictions are not always accurate and often indicate perfect software when more errors were known to exist. The predictions sometimes vacillate with the correction of a single additional error. For example, when using a sequence of 3 MTTFs (Appendix B), the predicted N at one point shows 4, then perfect twice in a row, then 4, then 18. The number of remaining errors in these predictions is, respectively, 1, 0, 0, 1, 15.

The predicted initial error content of 4, when using corrections #5 - #7 or #8 - #10, seems to be good, as this indicates one error left. (The LIC program at these points has, respectively, five and then two more known errors.) Generally, the JM model is not this accurate.

The overall weakness of this model is possibly the result of the existence of the large and small error(s) mentioned above, and the relatively small number of known errors (twelve) in the software. It should be noted that, in Appendices B and D, when sequences of the first X calculated MTTFs are used, the software is generally deemed to be free of any more errors. In real testing, these are the sequences of inter-time failures most likely to be used.

THE GEOMETRIC DE-EUTROPHICATION MODEL

The Moranda Geometric De-Eutrophication model performed better. Also, as might be expected, increasing the number of test cases used to establish the MTTF sequence greatly effects the results and the accuracy of the predictions.

In Appendix C, which includes error #1, the effect of the large first error is paramount. Initially for one million runs (with only two MTTFs used as input) a MTTF is predicted that is 5.29 times as great as the actual number derived empirically. This ratio of estimated MTTF and actual MTTF declines as more

data is used; this can be partly accounted for by the uniformity of the rate at which the software is improved. The closest this model comes to matching the estimated MTTF to the actual MTTF is when eight data points are entered (ie., when nine errors have been removed). This ratio is 0.861. Thereafter, near the end of the input of available data, the predictions are again skewed upward, when data from consecutive very small errors is used in the predictions.

Upon removing the effects of error #1 entirely from the initial software, the Geometric De-Eutrophication predictions of MTTF (found in Appendix E) improve greatly. Even runs consisting of only ten thousand inputs show rather remarkable estimates. The more accurate results obtained at one million inputs are used to calculate the numbers below. Prior to inserting the corrections of the last four small errors, the prediction is never wrong by more than 27 percent. (It predicts 127 percent of the actual MTTF at 5 data points.) The one poor prediction in this range is the .0151 ratio (1159 / 76,923) at 6 inputs. The next paragraph explains why it is not accurate in this case. The most accurate prediction is actually made with just the first two MTTFs: this ratio is just 1.0247 (63.95 / 62.41).

The De-Eutrophication model at one point seems to "correct itself". Note first, using the failure numbers from Appendix A under one million runs, that from Fix1_to_2 to Fix1_to_3 the correction is 47 percent ((56,927 - 29,833) / 56,927). The

succeeding corrections in percents are 46, 60, 52, 37 and then 99. This 99 percent correction is measured from 1966 failures at Fix123567_and_8 to 13 failures at Fix1_to_8 (ie., $(1966 - 13) / 1966$). The model misses this great correction (at 6 MTTFs entered) when it predicts a MTTF of 1,159 and the actual value is 76,923. However, its next prediction at 7 MTTFs, which now includes the seventh value (ie., the failure of only 13 times at Fix1_to_8), is much better. The model has gotten back on track with the additional input of the first of the very small errors, and the ratio of estimated MTTF and actual MTTF is .8369 ($104,612 / 125,000$). The remaining errors are small and the predictions for the rest of the figures are skewed upward a great deal, making their ratios very much larger than one.

The overall stability of the measure of failure rates against the gold version improves as the number of randomly generated inputs increases. This is not surprising; however, even between 100,000 and 1,000,000 runs, the difference in MTTF at Fix12356_And_8 (six errors taken out) is still greater than one. (It changes from 319.49 to 322.48.) The other five MTTFs before Fix12356_And_8 are at least stable to within plus or minus one. The failure rate of Fix1 (which will eventually stabilize at about 57.6%) moves from 100% to 40% to 60% from using one, ten and then one hundred inputs. Ten thousand runs are required to get this to stabilize to within one percentage point. And even the last 4 errors only begin to show up at one million runs. These changing MTTFs, which are as accurate as the number of runs

that were made, must have a degrading effect on the prediction of the Geometric De-Eutrophication model.

CONCLUSIONS

The Geometric De-Eutrophication model seems more adept at predicting the time to the next error, if it exists. The Jelinski / Moranda model did not perform well. Given the software used in this project, with only twelve known errors, the predictions were accurate in the Geometric model and might be acceptable in a real application. The small number of bugs with which to derive data and the results from that data is a credit to the Geometric De-Eutrophication model, but may have been a factor in hindering the ability of the JM model.

The elimination of a single abnormally large bug greatly improved the Geometric De-Eutrophication predictions. Predictions were skewed when this type of error was part of the set producing the MTTF data. When this software's large first error was removed, and the resulting version was used as the initial version, the predicted results were very good. It is easy to say that the De-Eutrophication model is more accurate when errors of irregular size are ignored, but it is difficult to determine when the software is ready to begin being predicted. Who is to say when abnormally large errors have ceased and those that will follow are decreasing at a uniform rate? If an error is classified as large, this does not say anything about the

errors that follow; they may all be "large", but decreasing at a uniform rate, as is desired for this model. Perhaps one should look for abrupt changes in the rate of increasing MTTFs as corrections are made.

Finally, it is generally true that the greater the number of inputs used in order to produce MTTF data, the more accurate the predictions will be. This is true throughout the different stages of the De-Eutrophication model as shown by the increasingly accurate MTTF predictions. The MTTFs used from Appendix A are, in some cases, only beginning to stabilize even at one million runs. In real-time testing, runs of over one million may be desirable. The number of inputs used to predict MTTFs might depend on the degree of accuracy needed in the application, and on the ability to continue to collect the next failure point and introduce it into the data set. At some point, comparing past predictions of MTTF (gained from software versions with fewer fixes) with those produced using the most current data available may help to determine when the predictions are accurate enough and the number of inputs used in finding the predictions is sufficiently high.

* *
* APPENDIX A *
* *

Results of random inputs on all software versions;
runs of 1; 10; 100; 1,000; 10,000; 100,000; 1,000,000.

	<u>SUCCESS</u>	<u>TOTAL FAILURES</u>	<u>TOTALS</u>
Fix1	0	1	1
Fix1_TO_2	0	1	1
Fix1_TO_3	0	1	1
Fix123_AND_5	0	1	1
Fix1235_AND_6	0	1	1
Fix12356_AND_8	1	0	1
Fix123567_AND_8	1	0	1
Fix1_TO_8	1	0	1
Fix1_TO_9	1	0	1
Fix1_TO_10	1	0	1
Fix1_TO_11	1	0	1
Fix1_TO_12	1	0	1

All Gold version results were 100% successful.

<u>VERSION</u>	<u>RELIABILITY</u>	<u>FAILURE RATE</u>	<u>MTTF</u>
Fix1	0.0000%	100.0000%	1.00
Fix1_And_2	0.0000%	100.0000%	1.00
Fix1_To_3	0.0000%	100.0000%	1.00
Fix123_And_5	0.0000%	100.0000%	1.00
Fix1235_And_6	0.0000%	100.0000%	1.00
Fix12356_And_8	100.0000%	0.0000%	N/A
Fix123567_And_8	100.0000%	0.0000%	N/A
Fix1_To_8	100.0000%	0.0000%	N/A
Fix1_To_9	100.0000%	0.0000%	N/A
Fix1_To_10	100.0000%	0.0000%	N/A
Fix1_To_11	100.0000%	0.0000%	N/A
Fix1_To_12	100.0000%	0.0000%	N/A

RESULTS OF 1 RANDOM INPUT

	<u>SUCCESS</u>	<u>TOTAL FAILURES</u>	<u>TOTALS</u>
	=====	=====	=====
Fix1	6	4	10
Fix1_TO_2	9	1	10
Fix1_TO_3	9	1	10
Fix123_AND_5	9	1	10
Fix1235_AND_6	9	1	10
Fix12356_AND_8	10	0	10
Fix123567_AND_8	10	0	10
Fix1_TO_8	10	0	10
Fix1_TO_9	10	0	10
Fix1_TO_10	10	0	10
Fix1_TO_11	10	0	10
Fix1_TO_12	10	0	10

All Gold version results were 100% successful.

<u>VERSION</u>	<u>RELIABILITY</u>	<u>FAILURE RATE</u>	<u>MTTF</u>
=====	=====	=====	=====
Fix1	60.0000%	40.0000%	2.50
Fix1_And_2	90.0000%	10.0000%	10.00
Fix1_To_3	90.0000%	10.0000%	10.00
Fix123_And_5	90.0000%	10.0000%	10.00
Fix1235_And_6	90.0000%	10.0000%	10.00
Fix12356_And_8	100.0000%	0.0000%	N/A
Fix123567_And_8	100.0000%	0.0000%	N/A
Fix1_To_8	100.0000%	0.0000%	N/A
Fix1_To_9	100.0000%	0.0000%	N/A
Fix1_To_10	100.0000%	0.0000%	N/A
Fix1_To_11	100.0000%	0.0000%	N/A
Fix1_To_12	100.0000%	0.0000%	N/A

RESULTS OF 10 RANDOM INPUTS

	SUCCESS =====	TOTAL FAILURES =====	TOTALS =====
Fix1	39	61	100
Fix1_TO_2	96	4	100
Fix1_TO_3	97	3	100
Fix123_AND_5	99	1	100
Fix1235_AND_6	99	1	100
Fix12356_AND_8	100	0	100
Fix123567_AND_8	100	0	100
Fix1_TO_8	100	0	100
Fix1_TO_9	100	0	100
Fix1_TO_10	100	0	100
Fix1_TO_11	100	0	100
Fix1_TO_12	100	0	100

All Gold version results were 100% successful.

VERSION =====	RELIABILITY =====	FAILURE RATE =====	MTTF =====
Fix1	39.0000%	61.0000%	1.64
Fix1_And_2	96.0000%	4.0000%	25.00
Fix1_To_3	97.0000%	3.0000%	33.33
Fix123_And_5	99.0000%	1.0000%	100.00
Fix1235_And_6	99.0000%	1.0000%	100.00
Fix12356_And_8	100.0000%	0.0000%	N/A
Fix123567_And_8	100.0000%	0.0000%	N/A
Fix1_To_8	100.0000%	0.0000%	N/A
Fix1_To_9	100.0000%	0.0000%	N/A
Fix1_To_10	100.0000%	0.0000%	N/A
Fix1_To_11	100.0000%	0.0000%	N/A
Fix1_To_12	100.0000%	0.0000%	N/A

RESULTS OF 100 RANDOM INPUTS

	<u>SUCCESS</u> =====	<u>TOTAL FAILURES</u> =====	<u>TOTALS</u> =====
Fix1	430	570	1,000
Fix1_TO_2	947	53	1,000
Fix1_TO_3	968	32	1,000
Fix123_AND_5	989	11	1,000
Fix1235_AND_6	996	4	1,000
Fix12356_AND_8	999	1	1,000
Fix123567_AND_8	1,000	0	1,000
Fix1_TO_8	1,000	0	1,000
Fix1_TO_9	1,000	0	1,000
Fix1_TO_10	1,000	0	1,000
Fix1_TO_11	1,000	0	1,000
Fix1_TO_12	1,000	0	1,000

All Gold version results were 100% successful.

<u>VERSION</u> =====	<u>RELIABILITY</u> =====	<u>FAILURE RATE</u> =====	<u>MTTF</u> =====
Fix1	43.0000%	57.0000%	1.75
Fix1_And_2	94.7000%	5.3000%	18.87
Fix1_To_3	96.8000%	3.2000%	31.25
Fix123_And_5	98.9000%	1.1000%	90.91
Fix1235_And_6	99.6000%	0.4000%	250.00
Fix12356_And_8	99.9000%	0.1000%	1,000.00
Fix123567_And_8	100.0000%	0.0000%	N/A
Fix1_To_8	100.0000%	0.0000%	N/A
Fix1_To_9	100.0000%	0.0000%	N/A
Fix1_To_10	100.0000%	0.0000%	N/A
Fix1_To_11	100.0000%	0.0000%	N/A
Fix1_To_12	100.0000%	0.0000%	N/A

RESULTS OF 1,000 RANDOM INPUTS

	<u>SUCCESS</u> =====	<u>TOTAL FAILURES</u> =====	<u>TOTALS</u> =====
Fix1	4,269	5,731	10,000
Fix1_TO_2	9,450	550	10,000
Fix1_TO_3	9,711	189	10,000
Fix123_AND_5	9,850	150	10,000
Fix1235_AND_6	9,945	55	10,000
Fix12356_AND_8	9,978	22	10,000
Fix123567_AND_8	9,986	14	10,000
Fix1_TO_8	10,000	0	10,000
Fix1_TO_9	10,000	0	10,000
Fix1_TO_10	10,000	0	10,000
Fix1_TO_11	10,000	0	10,000
Fix1_TO_12	10,000	0	10,000

All Gold version results were 100% successful.

<u>VERSION</u> =====	<u>RELIABILITY</u> =====	<u>FAILURE RATE</u> =====	<u>MTTF</u> =====
Fix1	42.6900%	57.3100%	1.74
Fix1_And_2	94.5000%	5.5000%	18.18
Fix1_To_3	97.1100%	2.8900%	34.60
Fix123_And_5	98.5000%	1.5000%	66.67
Fix1235_And_6	99.4500%	0.5500%	181.82
Fix12356_And_8	99.7800%	0.2200%	454.55
Fix123567_And_8	99.8600%	0.1400%	714.29
Fix1_To_8	100.0000%	0.0000%	N/A
Fix1_To_9	100.0000%	0.0000%	N/A
Fix1_To_10	100.0000%	0.0000%	N/A
Fix1_To_11	100.0000%	0.0000%	N/A
Fix1_To_12	100.0000%	0.0000%	N/A

RESULTS OF 10,000 RANDOM INPUTS

	<u>SUCCESS</u>	<u>TOTAL FAILURES</u>	<u>TOTALS</u>
Fix1	42,342	57,658	100,000
Fix1_TO_2	94,258	5,742	100,000
Fix1_TO_3	97,011	2,989	100,000
Fix123_AND_5	98,414	1,586	100,000
Fix1235_AND_6	99,352	648	100,000
Fix12356_AND_8	99,687	313	100,000
Fix123567_AND_8	99,809	191	100,000
Fix1_TO_8	99,999	1	100,000
Fix1_TO_9	100,000	0	100,000
Fix1_TO_10	100,000	0	100,000
Fix1_TO_11	100,000	0	100,000
Fix1_TO_12	100,000	0	100,000

All Gold version results were 100% successful.

<u>VERSION</u>	<u>RELIABILITY</u>	<u>FAILURE RATE</u>	<u>MTTF</u>
Fix1	42.3420%	57.6580%	1.73
Fix1_And_2	94.2580%	5.7420%	17.42
Fix1_To_3	97.0110%	2.9890%	33.46
Fix123_And_5	98.4140%	1.5860%	63.05
Fix1235_And_6	99.3520%	0.6480%	154.32
Fix12356_And_8	99.6870%	0.3130%	319.49
Fix123567_And_8	99.8090%	0.1910%	523.56
Fix1_To_8	99.9990%	0.0010%	100,000.00
Fix1_To_9	100.0000%	0.0000%	N/A
Fix1_To_10	100.0000%	0.0000%	N/A
Fix1_To_11	100.0000%	0.0000%	N/A
Fix1_To_12	100.0000%	0.0000%	N/A

RESULTS OF 100,000 RANDOM INPUTS

	<u>SUCCESS</u> =====	<u>TOTAL FAILURES</u> =====	<u>TOTALS</u> =====
Fix1	424,327	575,673	1,000,000
Fix1_TO_2	943,073	56,927	1,000,000
Fix1_TO_3	970,167	29,833	1,000,000
Fix123_AND_5	983,977	16,023	1,000,000
Fix1235_AND_6	993,522	6,478	1,000,000
Fix12356_AND_8	996,899	3,101	1,000,000
Fix123567_AND_8	998,035	1,966	1,000,001
Fix1_TO_8	999,988	13	1,000,001
Fix1_TO_9	999,992	8	1,000,000
Fix1_TO_10	999,994	6	1,000,000
Fix1_TO_11	999,993	7	1,000,000
Fix1_TO_12	999,994	6	1,000,000

All Gold version results were 100% successful.
(Runs of one million one are not misprints.)

<u>VERSION</u> =====	<u>RELIABILITY</u> =====	<u>FAILURE RATE</u> =====	<u>MTTF</u> =====
Fix1	42.4327%	57.5673%	1.74
Fix1_And_2	94.3073%	5.6927%	17.57
Fix1_To_3	97.0167%	2.9833%	33.52
Fix123_And_5	98.3977%	1.6023%	62.41
Fix1235_And_6	99.3522%	0.6478%	154.37
Fix12356_And_8	99.6899%	0.3101%	322.48
Fix123567_And_8	99.8034%	0.1966%	508.65
Fix1_To_8	99.9987%	0.0013%	76,923.15
Fix1_To_9	99.9992%	0.0008%	125,000.00
Fix1_To_10	99.9994%	0.0006%	166,666.67
Fix1_To_11	99.9993%	0.0007%	142,857.14
Fix1_To_12	99.9994%	0.0006%	166,666.67

RESULTS OF 1,000,000 RANDOM INPUTS

* *
* APPENDIX B *
* *

Jelinski / Moranda results.
One million inputs.
Includes error #1.

Inter-time failures entered =====	Predicted initial error content =====
first 2	Software now perfect.
first 3	Software now perfect.
first 4	Software now perfect.
first 5	Software now perfect.
first 6	Software now perfect.
first 7	Software now perfect.
first 8	Software now perfect.
first 9	Software now perfect.
first 10	Software now perfect.
first 11	Software now perfect.
first 12	25

Values tested: first X calculated MTTFs.

Inter-time failures entered =====	Predicted initial error content =====
1 - 3	Software now perfect.
2 - 4	Software now perfect.
3 - 5	Software now perfect.
4 - 6	Software now perfect.
5 - 7	4
6 - 8	Software now perfect.
7 - 9	Software now perfect.
8 - 10	4
9 - 11	18
10 - 12	Software now perfect.

Values tested: sequences of 3 MTTF.

Inter-time failures entered =====	Predicted initial error content =====
1 - 5	Software now perfect.
2 - 6	Software now perfect.
3 - 7	Software now perfect.
4 - 8	Software now perfect.
5 - 9	Software now perfect.
6 - 10	Software now perfect.
7 - 11	6
8 - 12	10

Values tested: sequences of 5 MTTF.

*
* APPENDIX C *
*

Moranda Geometric De-Eutrophication results.
Includes error #1.

Inter-time failures entered	Predicted initial error content
=====	=====
1 - 7	Software now perfect.
2 - 8	Software now perfect.
3 - 9	Software now perfect.
4 - 10	Software now perfect.
5 - 11	Software now perfect.
6 - 12	8

Values tested: sequences of 7 MTTF.

Number of random inputs: 1

First n calculated MTTFs	k	D	lambda (est.)	MTTF (est.)	MTTF (actual)
=====	=====	=====	=====	=====	=====
2	.9994	1.000031	.999909	1	1
3	.9994	1.000061	.99988	1	1
4	.9994	1.000092	.99985	1	1
5	.9994	1.000122	.99982	1	*

(* Did not fail in next 1 case.)

Number of random inputs: 10

First n calculated MTTFs	k	D	lambda (est.)	MTTF (est.)	MTTF (actual)
=====	=====	=====	=====	=====	=====
2	.25001	.399989	.02500	40	10
3	.49999	.300007	.03750	26.67	10
4	.68527	.231659	.05109	19.57	10
5	.79366	.194533	.06126	16.32	*

(* Did not fail in next 10 cases.)

Number of random inputs: 100

First n calculated MTTFs	k	D	lambda (est.)	MTTF (est.)	MTTF (actual)
=====	=====	=====	=====	=====	=====
2	.06558	.609830	.00262	381.24	33.33
3	.22182	.339927	.00371	269.54	100
4	.31629	.249286	.00249	400.82	100
5	.42163	.173845	.00232	431.71	*

(* Did not fail in next 100 cases.)

Number of random inputs: 1,000

First n calculated MTTFs	k	D	lambda (est.)	MTTF (est.)	MTTF (actual)
=====	=====	=====	=====	=====	=====
2	.09268	.571610	.00491	203.67	31.25
3	.23659	.376715	.00499	200.45	90.91
4	.30970	.300937	.00277	361.21	250
5	.33619	.273620	.00118	851.03	1,000
6	.32740	.284330	.00035	2,855.71	*

(* Did not fail in next 1,000 cases.)

Number of random inputs: 10,000

First n calculated MTTFs	k	D	lambda (est.)	MTTF (est.)	MTTF (actual)
=====	=====	=====	=====	=====	=====
2	.09573	.574644	.00527	189.88	34.60
3	.22426	.396975	.00448	223.35	66.67
4	.33375	.282887	.00351	284.92	181.82
5	.37012	.249989	.00174	575.93	454.55
6	.38440	.236102	.00076	1,312.81	714.29
7	.41040	.208959	.00041	2,440.62	*

(* Did not fail in next 10,000 cases.)

Number of random inputs: 100,000

First n calculated MTTFs	k	D	lambda (est.)	MTTF (est.)	MTTF (actual)
=====	=====	=====	=====	=====	=====
2	.09927	.578148	.00570	175.51	33.46
3	.22743	.404169	.00475	210.32	63.05
4	.33729	.289201	.00374	267.18	154.32
5	.38232	.248733	.00203	492.16	319.49
6	.40918	.224134	.00105	950.65	523.56
7	.43725	.197623	.00060	1,655.95	100,000.00
8	.24635	.516901	.000007	142,606.96	*

(* Did not fail in next 100,000 cases.)

Number of random inputs: 1,000,000

First n calculated MTTFs	k	D	lambda (est.)	MTTF (est.)	MTTF (actual)
=====	=====	=====	=====	=====	=====
2	.09903	.574725	.00564	177.43	33.52
3	.22780	.400969	.00474	210.98	62.41
4	.33948	.285518	.00379	263.69	154.37
5	.38379	.246333	.00205	487.54	322.48
6	.40954	.223023	.00105	950.27	508.65
7	.43872	.195770	.00061	1,632.95	76,923.15
8	.25709	.486989	.0000093	107,585.69	125,000.00
9	.25502	.494283	.0000023	443,487.55	166,666.67
10	.25783	.527950	.0000007	1,459,303.34	142,857.14
11	.27052	.457094	.0000003	3,853,062.31	166,666.67
12	.28529	.375454	.0000001	9,162,777.37	*

(* Did not fail in next 1,000,000 cases.)

* *
* APPENDIX D *
* *

Jelinski / Moranda results.
One million inputs.
Excludes error #1.

Inter-time
failures entered
=====

Predicted initial
error content
=====

first 2	3
first 3	Software now perfect.
first 4	Software now perfect.
first 5	Software now perfect.
first 6	Software now perfect.
first 7	Software now perfect.
first 8	Software now perfect.
first 9	Software now perfect.
first 10	Software now perfect.
first 11	Software now perfect.

Values tested: first X calculated MTTFs.

*
* APPENDIX E *
*

Moranda Geometric De-Eutrophication results.
Excludes error #1.

Number of random inputs: 1

First n calculated MTTFs	k	D	lambda (est.)	MTTF (est.)	MTTF (actual)
=====	=====	=====	=====	=====	=====
2	.9994	1.000031	.99991	1	1
3	.9994	1.000061	.99988	1	1
4	.9994	1.000092	.99985	1	*

(* Did not fail in next 1 case.)

Number of random inputs: 10

First n calculated MTTFs	k	D	lambda (est.)	MTTF (est.)	MTTF (actual)
=====	=====	=====	=====	=====	=====
2	.99994	.100003	.09999	10	10
3	.99994	.100006	.09999	10	10
4	.99994	.100009	.09998	10	*

(* Did not fail in next 10 cases.)

Number of random inputs: 100

First n calculated MTTFs	k	D	lambda (est.)	MTTF (est.)	MTTF (actual)
=====	=====	=====	=====	=====	=====
2	.75009	.040000	.02250	44.43	100
3	.49999	.045002	.00562	177.78	100
4	.58555	.040453	.00476	210.28	*

(* Did not fail in next 100 cases.)

Number of random inputs: 1,000

First n calculated MTTFs	k	D	lambda (est.)	MTTF (est.)	MTTF (actual)
=====	=====	=====	=====	=====	=====
2	.60386	.052993	.01932	51.75	90.91
3	.45556	.057722	.00546	183.24	250.00
4	.41577	.061026	.00182	548.37	1,000.00
5	.36817	.067984	.00046	2,174.56	*

(* Did not fail in next 1,000 cases.)

Number of random inputs: 10,000

First n calculated MTTFs	k	D	lambda (est.)	MTTF (est.)	MTTF (actual)
=====	=====	=====	=====	=====	=====
2	.52538	.055008	.01518	65.86	66.67
3	.52220	.055118	.00785	127.41	181.82
4	.46862	.059044	.00285	351.19	454.55
5	.44518	.061963	.00108	922.92	714.29
6	.46032	.059323	.00056	1,771.8	*

(* Did not fail in next 10,000 cases.)

Number of random inputs: 100,000

First n calculated MTTFs	k	D	lambda (est.)	MTTF (est.)	MTTF (actual)
=====	=====	=====	=====	=====	=====
2	.52062	.057406	.01556	64.27	63.05
3	.52562	.057223	.00831	120.34	154.32
4	.48742	.060080	.00339	294.90	319.49
5	.47973	.061020	.00155	645.00	523.56
6	.49352	.058754	.00085	1,177.97	100,000.00
7	.24843	.122359	.00001	139,949.23	*

(* Did not fail in next 100,000 cases.)

Number of random inputs: 1,000,000

First n calculated MTTFs	k	D	lambda (est.)	MTTF (est.)	MTTF (actual)
=====	=====	=====	=====	=====	=====
2	.52416	.056916	.01564	63.95	62.41
3	.53063	.056680	.00847	118.09	154.37
4	.48913	.059736	.00342	292.47	322.48
5	.47973	.060875	.00155	646.53	508.65
6	.49547	.058309	.00086	1,159.16	76,923.15
7	.26027	.118164	.0000096	104,612.75	125,000.00
8	.25758	.119670	.0000023	431,217.65	166,666.67
9	.26661	.112449	.0000008	1,306,476.59	142,857.14
10	.28004	.100866	.0000003	3,342,340.95	166,666.67
11	.29774	.082426	.0000001	7,442,966.15	*

(* Did not fail in next 1,000,000 cases.)

```
*****
*                                     *
*      APPENDIX F                    *
*                                     *
*****
```

Two pascal programs.
Jelinski / Moranda reliability model.
Moranda De-Eutrophication reliability model.

```

{*****}
Christopher Cowles
ODU Master's Project
Summer, 1991
Reliability model by Jelinski & Moranda
{*****}
program jm (input, output);

const max_try = 1000;    {try this many points beyond N}
      data_points = 11;  {max points for this model}

var   N,          {initial error content}
      m :         {number of inter-failure times entered}
      integer;

Time : array [1..data_points] of real;  {time intervals between
                                         successive errors}

T,          {sum of the elements of array Time}
RHS,        {current value of right hand side}
LHS,        {current value of left hand side}
sigma :     {sum from i=1 to m of (i-1) * Time sub_i}
real;

lhs_greater, {set true if LHS is greater}
rhs_greater, {set true if RHS is greater}
crossed :    {initialized to false; ends loop if set to true}
boolean;

{*****}
Function find_sigma
- This procedure calculates the value of the constant (i-1)
  times Time sub_i.
{*****}
function find_sigma : real;

var i : integer;
    temp : real;

begin
  temp := 0.0;
  for i := 1 to m do
    temp := temp + ((i-1) * (Time[i]));
  find_sigma := temp;
end;

```

```

{*****}
  Procedure Init
  - This procedure reads in the values from the keyboard,
    sets the value of T, sigma, and the 3 booleans.
{*****}
procedure init;

var i : integer;

begin
  writeln ('How many different time intervals to be entered?');
  readln (m);
  writeln ('Enter the', m:3, ' values, pressing <ENTER> each time. ');
  for i := 1 to m do
    readln (Time[i]); {read data into Time array}

  N := m;
  lhs_greater := false;
  rhs_greater := false;
  crossed := false;
  sigma := find_sigma;

  T := 0.0;
  for i := 1 to m do
    T := T + Time[i]; {sum the elements of array Time}
end;

{*****}
  Function get_left_side
  - This procedure calculates the value of the left hand
    side of the equation.
{*****}
function get_left_side : real;

var i : integer;
    temp : real;

begin
  temp := 0.0;
  for i := 1 to m do
    temp := temp + (1/(N-(i-1)));

  get_left_side := temp;
end;

```

```

{*****}
Function get_right_side
- This procedure calculates the value of the right hand
  side of the equation.
{*****}
function get_right_side : real;

begin
  get_right_side := m/(N - ((1/T) * sigma));
end;

{*****}
Procedure Compare
- This procedure determines if the two values (RHS & LHS)
  have crossed in value.
{*****}
procedure compare;

begin
  if rhs_greater and (LHS >= RHS)
  then crossed := true
  else if lhs_greater and (RHS >= LHS)
  then crossed := true;
  if crossed then writeln ('The values have crossed at ', N:5);
end;

{*****}
Procedure do_first_compare
- This procedure does the first comparison of LHS & RHS
  at the first value of N.
{*****}
procedure do_first_compare;

begin
  LHS := get_left_side;
  RHS := get_right_side;
  if LHS > RHS then lhs_greater := true
  else if RHS > LHS then rhs_greater := true
  else begin
    crossed := true;
    writeln ('Values are initially equal at ', N:4);
  end;
end;

```

```
{*****  
      MAIN MODULE  
*****}
```

```
begin  
  init;  
  do first_compare;  
  while (N < max_try) and (not(crossed)) do  
    begin  
      N := N + 1;  
      LHS := get_left_side;  
      RHS := get_right_side;  
      compare;  
    end;  
    if not (crossed)  
      then writeln ('Values never cross up to ', max_try:5);  
  end.
```

```
-----  
Pascal program for Jelinski / Moranda model  
-----
```



```

{*****
Christopher Cowles
ODU Master's Project
Summer, 1991
Geometric De-Eutrophication reliability
model by Moranda
*****}
program de_eutrophication (input, output);

const
  step = 0.0001;      {get this accurate}
  data_points = 11;
  max_right = 1.0;    {maximum value of k}

var
  m : integer;      {successive number of failures}

  RHS,              {current value of right hand side}
  LHS,              {current value of left hand side}
  test_point,      {test this value, between 0 and 1}
  left,            {value of left border}
  right:           {value of right border}
    real;

  LHS_greater_on_left,
  RHS_greater_on_right,
  LHS_greater_on_right,
  RHS_greater_on_left:
    boolean;

  Time : array [1..data_points] of real;

{*****
Function power
- This function calculates x to the n power.
*****}
function power (x, n : real) : real;

begin
  power := exp (n * (ln(x)));
end;

{*****
Function get_test_point
- This function sends back a point midway between the
current values of left and right.
*****}
function get_test_point : real;

```

```

begin
  get_test_point := ((right - left)/2) + left;
end;

{*****
  Procedure reassign_boundary
  - This procedure determines if the left or right hand side should be
    set equal to the current test_point. Boundary should not cross
    over the point where the LHS and RHS themselves cross between 0
    and 1.
  *****/}
procedure reassign_boundary;

begin
  if LHS_greater_on_left and (LHS > RHS)
  then left := test_point
  else if RHS_greater_on_left and (RHS > LHS)
  then left := test_point
  else if LHS_greater_on_right and (LHS > RHS)
  then right := test_point
  else if RHS_greater_on_right and (RHS > LHS)
  then right := test_point
  else writeln ('ERROR in reassign_boundary at ', test_point:5:4);
    {this last stmt. is a flag}
end;

{*****
  Procedure Evaluate
  - This procedure evaluates LHS and RHS with the value
    of k (between 0 and 1) that is sent to it.
  *****/}
procedure evaluate (k:real);

var i : integer;
    numerator,
    denominator : real;

begin
  LHS := (m+1) / 2;
  numerator := 0.0;
  for i := 1 to m do
    numerator := numerator + (i * (power(k,i)) * Time[i]);

  denominator := 0.0;
  for i := 1 to m do
    denominator := denominator + ((power(k,i)) * Time[i]);
  RHS := numerator/denominator;
end;

```

```

{*****
Procedure Init
- This procedure reads in the values from the keyboard,
  sets the value of T, sigma, and the 3 booleans.
*****}
procedure init;

var i : integer;

begin
  writeln ('How many different values to be entered?');
  readln (m);
  writeln ('Enter the', m:3, ' values, pressing <RETURN> each time.');
```

for i := 1 to m do
 readln (Time[i]);

LHS_greater_on_left := false;
RHS_greater_on_right := false;
LHS_greater_on_right := false;
RHS_greater_on_left := false;
left := step; {start at min value of k}
right := max_right; {start at max value of k}

evaluate (left);
if LHS > RHS then LHS_greater_on_left := true
else RHS_greater_on_left := true;
evaluate (right);
if RHS > LHS then RHS_greater_on_right := true
else LHS_greater_on_right := true;
end;

{*****
Procedure do_report
- This procedure prints out the desired results.
*****}

```

procedure do_report;

var D,
    lambda, {failure rate}
    denominator {denominator of fraction to find D}
    : real;
    i : integer;

begin
  writeln ('The value of k is ', test_point:6:5);
  denominator := 0.0;
  for i := 1 to m do
    denominator := denominator + ((power(test_point, (i-1))) *
                                   Time[i]);
  D := m / denominator;
  writeln ('The value of D is ', D:8:6);

```

```

lambda := (D * power(test_point, m));
writeln ('The failure rate lambda is estimated at ',
        lambda:6:7);{6:5);}
writeln ('The MTTF is estimated at ', (1/lambda):10:2);
end;

{*****
  MAIN MODULE
  *****)

begin
  init;
  while ((right - left) > step) do
    begin
      test_point := get_test_point; {get new mid-point}
      evaluate (test_point);        {evaluate LHS and RHS}
      reassign_boundary;             {set either left or right to
                                     test_point}
    end;
  do_report;
end.

```

```

-----
Pascal program for Moranda Geometric De-Eutrophication model
-----

```

REFERENCES

1. Janet R. Dunham, "Experiments in Software Reliability: Life-Critical Applications", IEEE Transactions On Software Engineering, Vol SE-12, No. 1, January 1986.
2. Z. Jelinski and P. Moranda, "Software Reliability Research", Statistical Computer Performance Evaluation, Walter Freiberger, Ed. New York: Academic, 1972, pp. 465-484.
3. P. B. Moranda, "Prediction Of Software Reliability During Debugging", Proceedings 1975 Annual Reliability and Maintainability Symposium, pp. 327-332.
4. J. D. Musa and K. Okumoto, "A Logarithmic Poisson Execution Time Model for Software Reliability Measurement", 1984, IEEE.
5. Richard L. White and Christine F. Harbison, "The Error Graph Research In Software Reliability", (unpublished Old Dominion University master's project, December 1990).

Simulation Studies of The Basic Execution Time Model

by Keith J. Lirette

Abstract

The Basic Execution Time software reliability model was tested for reliability parameter estimation accuracy on simulated data from the distribution it is based on. The model performed well on all but the smallest sets of test data generated. Data replication was tested and proved to increase the accuracy of parameter estimations. Confidence intervals generated proved to be ineffective in providing an accurate range for estimates.

0. Introduction

Software reliability is the probability that a computer program will operate failure-free for a particular period of time in a particular environment [1]. A large portion of software reliability literature is aimed at reliability estimation. Mathematical software reliability models have been developed which attempt to provide estimations of software reliability via statistical methods. Generally, these models use failure time data of the software in question and make estimations of the total fault content of the software and the current software failure rate. The difference between models lies in the assumptions made about the probability distribution of program failure times and the relative effect each fault has on the total failure rate of the software.

This paper is devoted to the study of one software reliability model. The focus of the study is to determine how well the model estimates reliability parameters from data generated according to the assumptions made by the model (distribution of faults, ...). If the model makes accurate estimates, then it is up to the potential model user to determine if the assumptions made by the model are reasonable for the software in question. If the model fails to make accurate predictions, then the internal consistency of the model should be questioned.

The paper continues on to study the effect of data replication on estimates and the use of confidence intervals as a way of providing a probable range rather than a single point estimate.

1. The Model

The model studied is the Basic Execution Time Model developed by John Musa. According to Musa, this model "is the most thoroughly developed and has been most widely applied to actual projects." The model is characterized by an exponential failure intensity (with respect to time) and a non-homogeneous poisson failure distribution [1].

The model is based on the assumptions [1]:

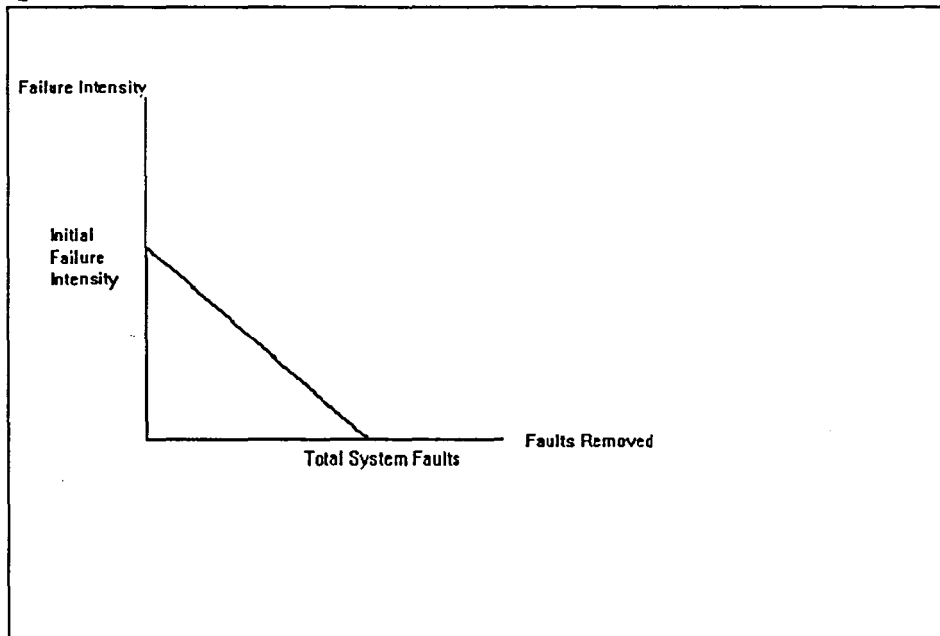
- 1) failures occur as a random process, which is a non-homogeneous poisson process,
- 2) a fixed failure intensity per fault (i.e. all faults contribute equally to the overall failure intensity),
- 3) fault corrections are immediate upon detection and perfect.

The failure intensity as a function of the number of faults removed is given by the function [1]:

$$\lambda(\mu) = \lambda_0 \left[1 - \frac{\mu}{\gamma_0} \right]$$

μ equals failures experienced, λ_0 is the initial failure intensity, and γ_0 total faults in the system.

The graph below shows the effect of removing faults on the failure intensity [1].



The failure intensity as a function of time is given by the exponential equation [1]:

$$\lambda(t) = \lambda_0 \exp\left[-\frac{\lambda_0}{\gamma_0} t\right]$$

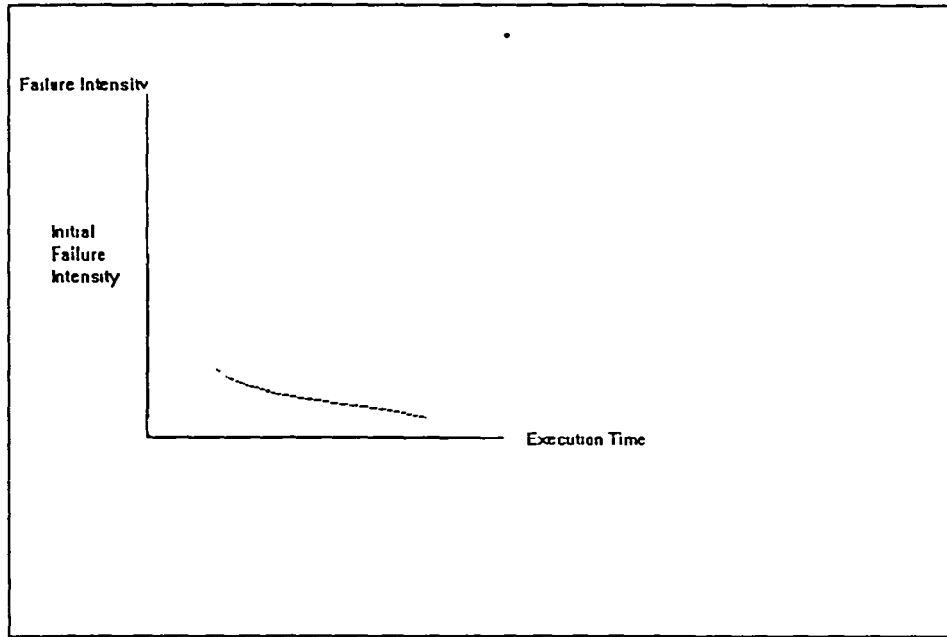
For generalization purposes, Musa reparameterizes this equation to [1]

$$\lambda(t) = \beta_0 \beta_1 \exp(-\beta_1 t)$$

where $\beta_0 = \gamma_0$, estimated total failures, and

$$\beta_1 = \frac{\lambda_0}{\gamma_0}, \text{ estimated failure rate per fault.}$$

The graph below shows an example of passing execution time on the failure intensity [1].



Parameter estimate equations are derived by the use of maximum likelihood techniques. Maximum likelihood can be used when the underlying distribution of the data is known. Maximum likelihood yields parameter estimates that make it most likely that the collected data occurred [1]. The mathematics is detailed in Appendix F. The maximum likelihood equations for B_0 and B_1 are given by [1]:

$$\beta_0 = \frac{m_e}{1 - \exp(-\beta_1 t_e)}$$

and [1]

$$\frac{m_e}{\beta_1} - \frac{m_e t_e}{\exp(\beta_1 t_e) - 1} - \sum_{i=1}^{m_e} t_i = 0$$

where m_e is the number of faults found and fixed, t_e is the time to end of testing, and t_i 's are the times to failure for each fault found.

2. Data Simulation and Estimation

The equations described in Section 1 provide the theory to base an estimation program upon. The first step in testing the model is the simulation of failure data. This was accomplished by using the distribution function of faults [1]:

$$F(t) = 1 - e^{-\int_{tsum}^{t+tsum} \lambda(x) dx}$$

where t = time, and $tsum$ is the sum of all previous failure times.

The model assumes that failures occur randomly. Thus we can randomly generate values between 0 and 1 for $F(t)$ and solve the right hand side of the distribution function for t (which is a simulated interfailure time). Repeating this, we can generate a set of failure times. The time variable t is solved for as

$$t = \frac{-\gamma_0}{\lambda_0} \ln \left(1 + \frac{\ln(1-r)}{\gamma_0 e^{-(\lambda_0/\gamma_0) tsum}} \right)$$

where, r is the randomly generated value for $F(t)$.

Implementing program solutions for the maximum likelihood equations covered in Section 1, estimates for the B_0 and B_1 parameters can be found for generated failure time sets. The program "BASIC.P" contained in Appendix G takes inputs for the total number of program faults, the failure rate per fault, the number of failure times to simulate, and the number of failure time sets to generate (50 was used throughout). The program generates the desired number of failure time sets and makes estimates for B_0 and B_1 .

3. Modeling Results

Parameters were chosen to produce failure data with. Three different values for the total number of program faults were used : 50, 100, and 200. A series of trials were run with each total fault count. The number of failure times generated started at 10% of the total fault count and was increased in 10% increments until 90% of the total program faults were generated (e.g. for 100 total program faults, 10, 20, ..., 90 trial sizes were used). Each trial consisted of estimates based on each of 50 failure time set generations (50 for 10 failures, 50 for 20 failures, etc.). The same per fault failure rate was used throughout, 0.001. Scatter plots were produced showing the difference between the estimates and the actual parameter value. Additionally, summary graphs were generated to allow for the detection of trends.

Appendix A contains the scatter plots and summary graphs for each of the trials. The scatter plots show the percent difference of each of the 50 calculated estimates of total program faults, B_0 , from the actual parameter value. An entry in a particular column

indicates that an estimate was within plus or minus 5% of that amount off of the actual parameter (e.g. an entry in the 10 column indicates that a particular estimate was between 5% and 15% off from the actual parameter). The summary graphs show the percentage off of the mean of the 50 calculated estimates from the actual parameter for each of the trials run for a particular total fault count.

For purpose of analysis I needed to define a measure of acceptability for the estimates. I chose to use this criteria: a set of estimates are acceptable if a significant majority of the estimates (2/3) are within plus or minus 30% of the actual parameter. The closer the estimates to the actual parameter, the better the estimates. This measure was arbitrarily chosen, but is illustrative of other possible criteria.

Using the criteria defined above, only when 90% of the total faults were generated was an acceptable set of estimates calculated for the 50 total fault size trials. The 100 fault trials reached acceptability at 60% of total faults generated, and **estimates continued to improve with greater number of faults generated**. The 200 fault trials also reached acceptability at 60% of total faults generated.

The 50 total fault size appears to provide too small of an event space to allow accurate estimation. Comparison of the space sizes for the 100 and 200 fault trials reveals that the estimates of the 200 fault case are more accurate. It is hypothesized that this is due to the larger number of failure times generated. For example, a 50% trial size for 100 total faults is 50 failure times, where a 50% trial size for 200 total faults is 100 failure times. Note that the expected time to find 50% of the faults is equal for all trial sizes (see t_0 mean in summary plots Appendix A). With the same per fault failure rate, a program with 200 faults will fail at double the frequency of a program with 100 faults. Thus, twice the number of failures are experienced in the same span of time.

It was also observed that with the same number of failure times generated, estimation for the smaller total fault count is more accurate. It is hypothesized that this is due to fact that removal of faults in the program with the smaller fault count has a larger effect on the residual program failure intensity, causing greater times between failure, and allowing for better statistical analysis.

Sample tests were performed with different per fault failure rates. The results scaled perfectly. That is, if 0.002 was used instead of 0.001, estimates for B_1 were exactly double, and estimates for B_0 were exactly the same.

4. Data Replication

Nagel [2] introduced the idea of data replication as a means to more accurately study the assumption that all faults have the same failure rate. Shen and Wilson [3] used data replication to test its effects on the accuracy of parameter estimation by the Jelinski-Moranda and Moranda Geometric reliability models. I also used it to test its effect on the accuracy of reliability parameter estimation. Instead of producing a single set of failure data, r replicates are made. This is synonymous with debugging the same program from its initial version r times. The corresponding failure times are averaged to obtain a single set of failure times. This average of the replicates is then used to estimate the reliability parameters.

Trials were run, each consisting of 50 separate failure time generations as before, but with each time set consisting of 2, 3, 4, 5, 10, 20, 30, ..., or 100 replications. Appendix B contains scatter plots and summary graphs like those in Appendix A, but for the 30 replicate data trials. Comparison of the scatter plots for no replication in Appendix A with those with 30 replications in Appendix B, displays the sharp improvement in estimation. For the 50 total fault trials, estimations become "acceptable" with 70% of faults generated and 30 replications versus 90% with no replication. For the 100 total fault trials, acceptability is reached with 50% of faults generated versus 60% without replication. And, for the 200 total fault trials, acceptability is reached with 40% of faults generated versus 60% without replication.

Appendix C contains summary graphs which show the effect of increasing the number of replications for a given number of failure times with each total fault count used. Each Appendix C graph shows the change in the B_0 parameter estimation for one trial size with the different number of data replications used. Study of the summary graphs shows that for the 50 total fault trials, parameter estimation improvement begins at 25 failure times produced and 20 data replications. Review of all the summary graphs indicates that **parameter estimation improvement is minimal after 30 replications.** The summary graphs behave asymptotically starting at 30 replications and continuing onto 100 replications. Replication does provide greater estimation improvement for smaller total fault counts. This is because smaller total fault counts provide less accurate estimations and thus have greater room for improvement. Still, with the same amount of replication and percentage of total faults produced, larger total fault counts yield more accurate estimates than smaller total fault counts.

What does replication provide? It removes some of the randomness of fault occurrence and provides a truer picture of the distribution of program faults. The figures contained in Appendix D, illustrate the gains from replication. It is shown that by increasing replication, fewer faults need to be located to achieve

the same accuracy of estimation as for fewer replications.

5. Confidence Intervals

Confidence intervals allow for generation of a range of parameter estimates that explain the data collected with some level of confidence. Musa provides the equations required to generate confidence intervals for B_0 and B_1 . Once a confidence interval is obtained for B_1 , the endpoints of its confidence interval can be substituted into the equation for B_0 to obtain a corresponding confidence interval for B_0 [1].

The upper/lower limits of an approximate 100(1- α) percent CI for β_1 are given by

$$\beta_1 \pm \frac{K_{1-\alpha/2}}{\sqrt{I(\beta_1)}}$$

where $K_{1-\alpha/2}$ is the appropriate normal deviate, and

$I(\beta_1)$ is the expected or Fisher information given by:

$$I(\beta_1) = m_e \left[\frac{1}{\beta_1^2} - \frac{t_e^2 \exp(\beta_1 t_e)}{[\exp(\beta_1 t_e) - 1]^2} \right]$$

One problem encountered was that the offset for the confidence interval around B_1 was often larger than the estimate for B_1 . Thus, the lower bound of the B_1 confidence interval was truncated at 0. But, this value was needed to calculate the upper bound of the confidence interval around B_0 . Zero could not be used in the equation to solve for B_0 (this would cause a zero divide). Additionally, testing showed that confidence intervals calculated with larger percents of the total faults generated (70, 80, 90%) did not provide the required confidence.

Appendix E contains summary graphs of 50, 55, ..., 95 percent confidence interval tests for the 3 total fault count sizes (50, 100, 200). Data was generated in the same manner as for the point estimation graphs: starting at 10% of the total faults and increasing by 10% until 90% of total faults were generated. 200 runs were made on each trial size and an appropriate confidence interval calculated for each. For each of the 200 confidence intervals calculated, it was determined whether the actual parameter value used to generate the estimate fell within it. The summary graphs show the percent of the 200 runs whose confidence interval contained the actual parameter value.

The accuracy of the confidence intervals, in relation to how many contained the actual parameter value, remained fairly constant between the different size total fault count trials. With smaller numbers of faults generated, parameter estimates were less accurate and confidence intervals were extremely large and useless. As parameter estimates improved with a greater number of total faults generated, confidence intervals tightened to a useable size. But,

these confidence intervals did not contain the required percentage of the actual parameters, and thus do not provide the expected confidence. As seen in the scatter plots of B_0 in Appendix A, the "acceptable" estimates tended to be greater than the actual parameter. Thus, confidence intervals around these greater values tended to miss high (i.e. the lower bound of the confidence interval was greater than the actual parameter).

6. Conclusion

The Basic Execution Time Model generally performed well on test data generated from the distribution it is based on. The exception was when a smaller number (50) of program faults were used, estimations were less accurate. Other total fault sizes performed well when at least 60% of the total faults were generated. Replication was found to be a means of increasing the accuracy of parameter estimations. With replication fewer faults needed to be found to get estimations as accurate as those estimated without replication. Finally, confidence intervals proved to be ineffective in providing a probable range for the estimated parameters.

References

1. J. Musa, A. Iannino, and K. Okumoto. Software Reliability: Measurement, Prediction, Application. New York: McGraw Hill, 1987.
2. Phyllis M. Nagel and James A. Skrivan, "Software Reliability: Repetitive Run Experimentation and Modeling", NASA Contractor Report 165836, Feb 1982.
3. Wenhui Shen and Larry Wilson, "Simulation Studies of Software Models", Old Dominion University Computer Science Department, Mar 1989.

Appendix A

This appendix contains two types of graphs showing the modeling results for single repetitions. The first graphs are scatter plot of trials for the different total fault counts used and number of failures generated. Each scatter plot graph shows how much the each of the 50 generated estimates for B_0 , total program faults, differ (within plus or minus 5%) from the actual parameter Γ_0 . n indicates the number of failures times produced, and ϕ is the per fault failure rate used.

The second graphs are summary graphs of the scatter plot graphs. Each summary graph shows how much the mean of the 50 calculated estimates for B_0 differs from the actual parameter Γ_0 for the different number of failures generated.

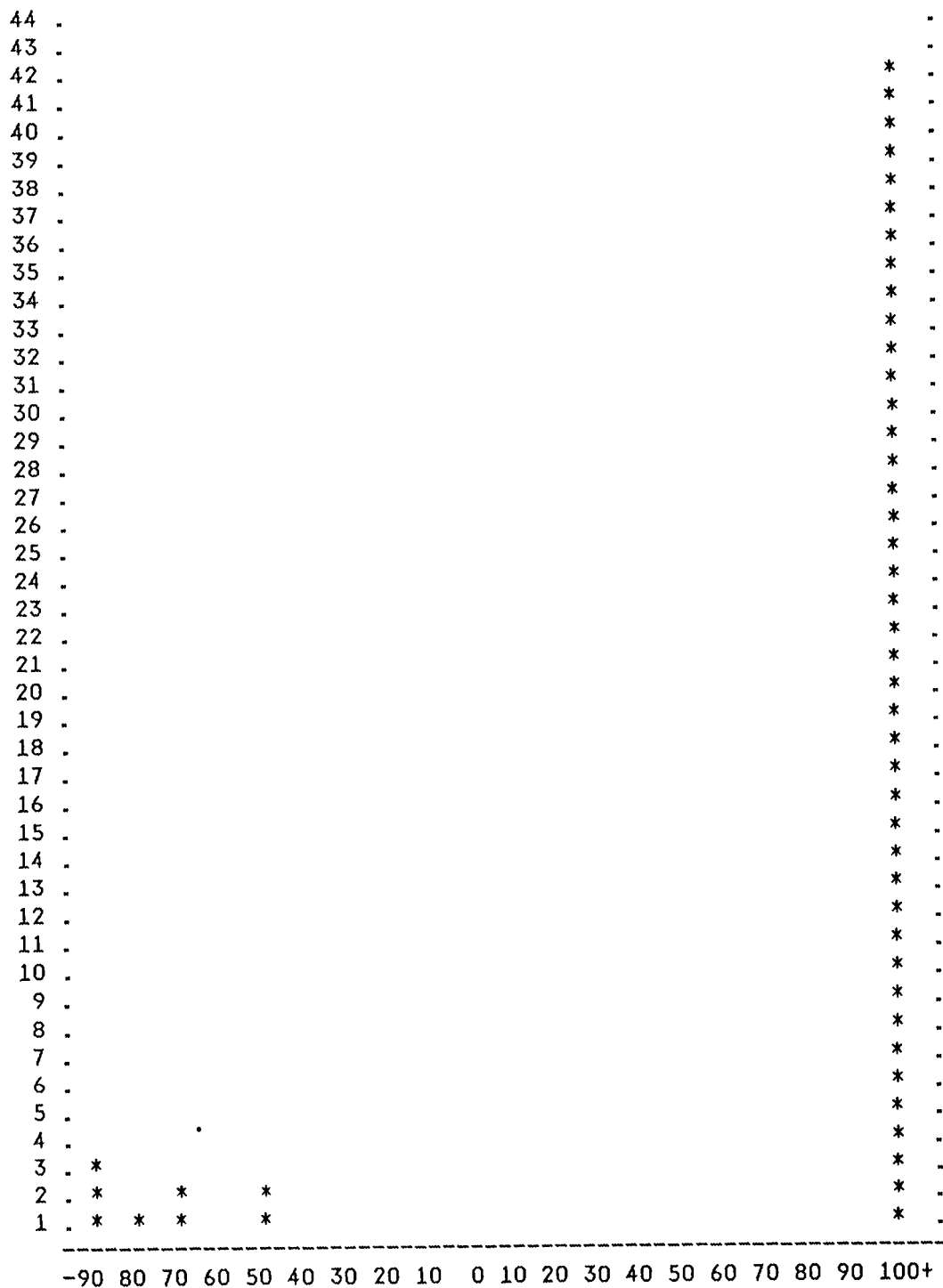
APPENDIX A INDEX

A1-1	Scatter Plots for 50 total faults
A2-1	Scatter Plots for 100 total faults
A3-1	Scatter Plots for 200 total faults
A4-1	Summary Graph for 50 total faults
A5-1	Summary Graph for 100 total faults
A6-1	Summary Graph for 200 total faults

Chart of Estimates of B_0 with :

$\Gamma_0 = 50$ $\Phi = 0.001$
 $n = 5$

Source : 50out01
 Output File : 50out01.his



% B_0 Differs from Γ_0

Chart of Estimates of B_0 with :

$\Gamma_0 = 50$ $\Phi = 0.001$
 $n = 10$

Source : 50out01
 Output File : 50out01.his

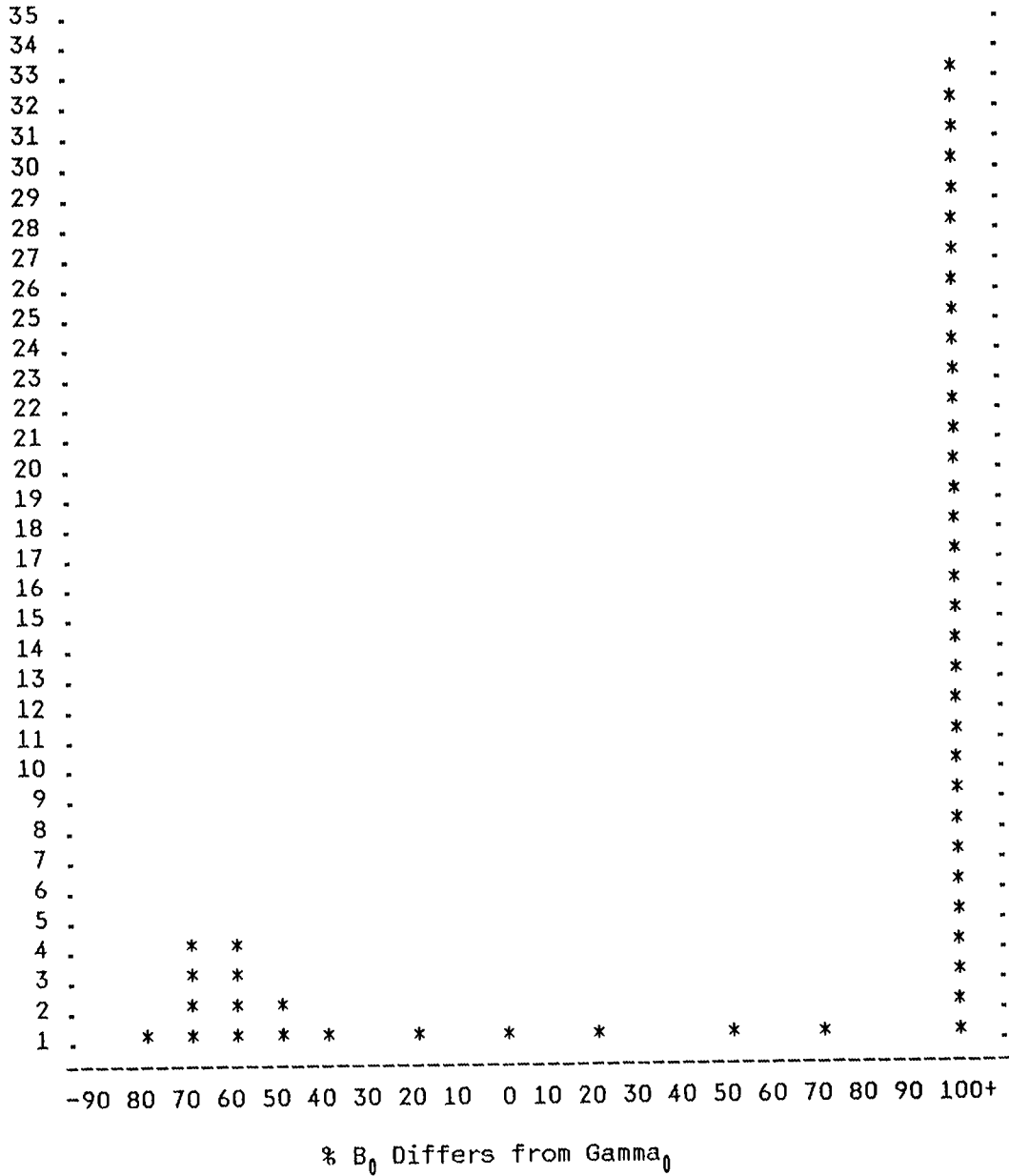
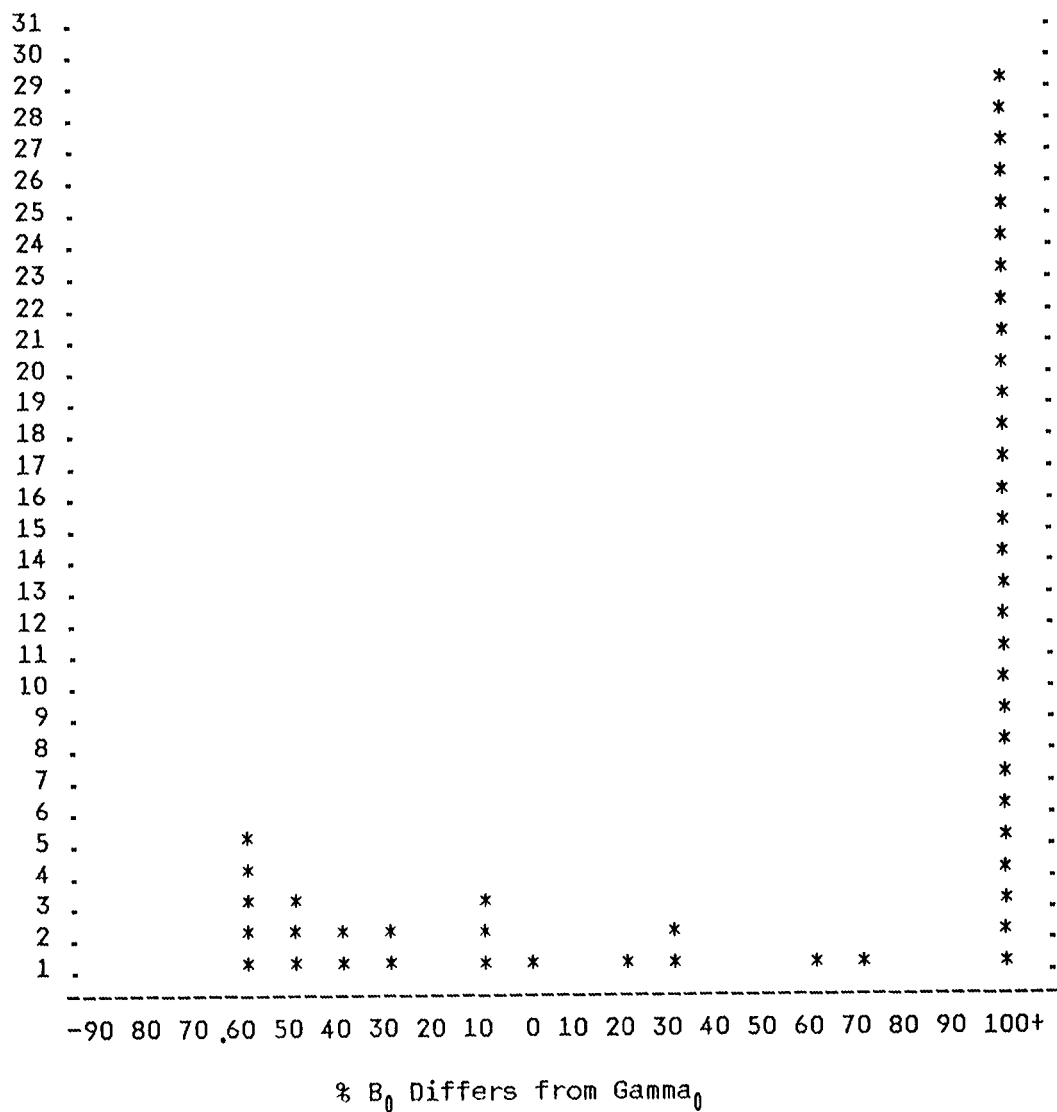


Chart of Estimates of B_0 with :

$\Gamma_0 = 50$ $\Phi = 0.001$
 $n = 15$

Source : 50out01
 Output File : 50out01.his



```
Source : 50out01
Output File : 50out01.his
```



Chart of Estimates of B_0 with :

Gamma₀ = 50 Phi = 0.001
 n = 25

Source : 50out01
 Output File : 50out01.his

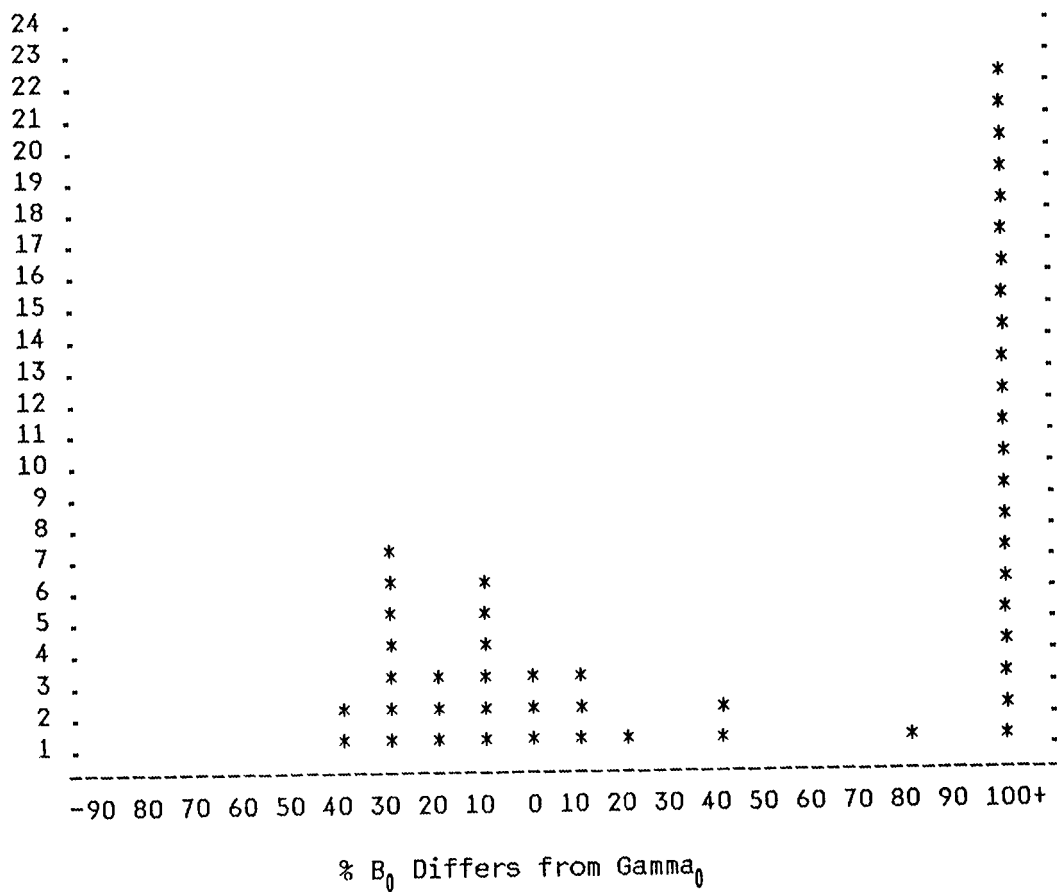


Chart of Estimates of B_0 with :

$\Gamma_0 = 50$ $\Phi = 0.001$
 $n = 30$

Source : 50out01
 Output File : 50out01.his

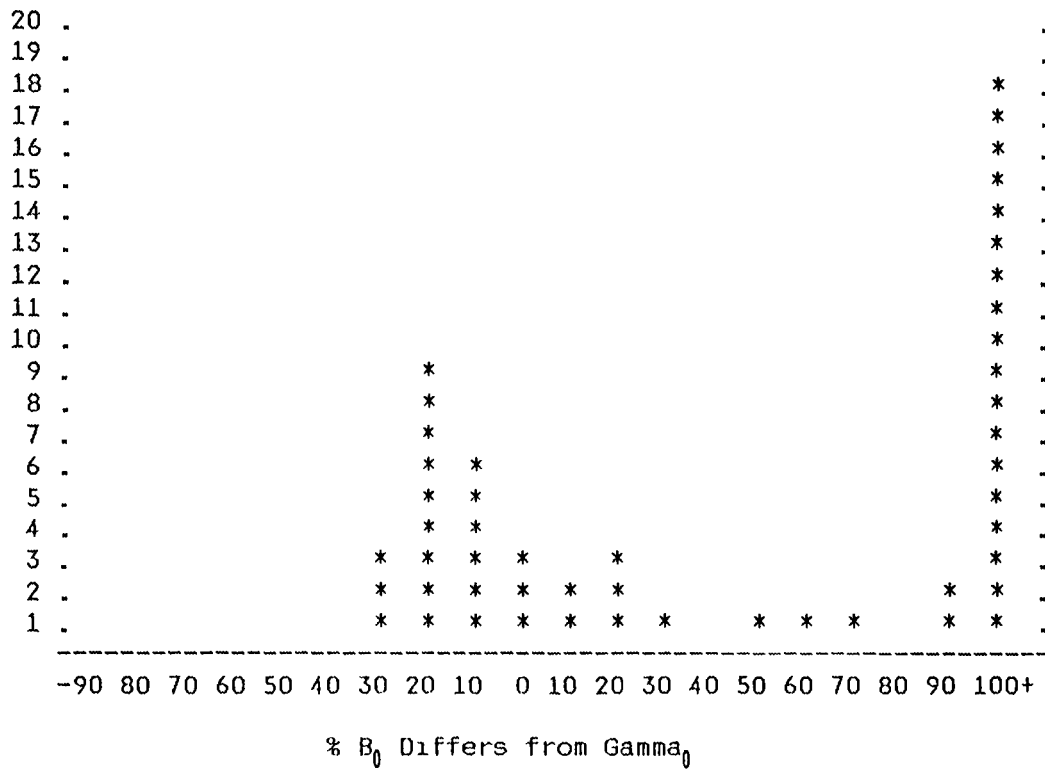


Chart of Estimates of B_0 with :

$\Gamma_0 = 50$ $\Phi_1 = 0.001$
 $n = 35$

Source : 50out01
 Output File : 50out01.his

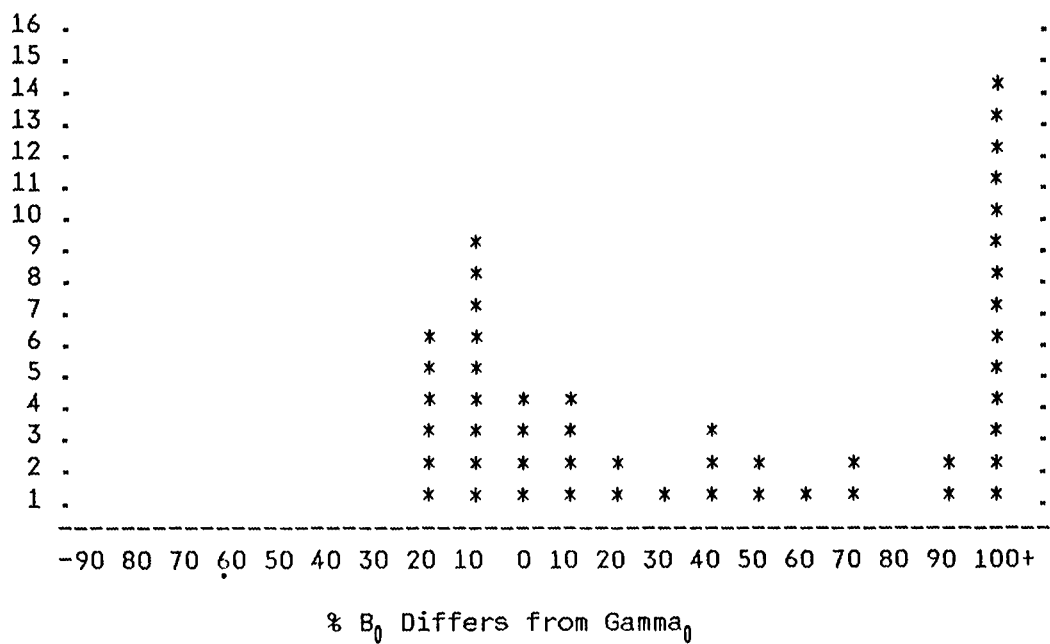


Chart of Estimates of B_0 with :

$\Gamma_0 = 50$ $\Phi = 0.001$
 $n = 40$

Source : 50out01
 Output File : 50out01.his

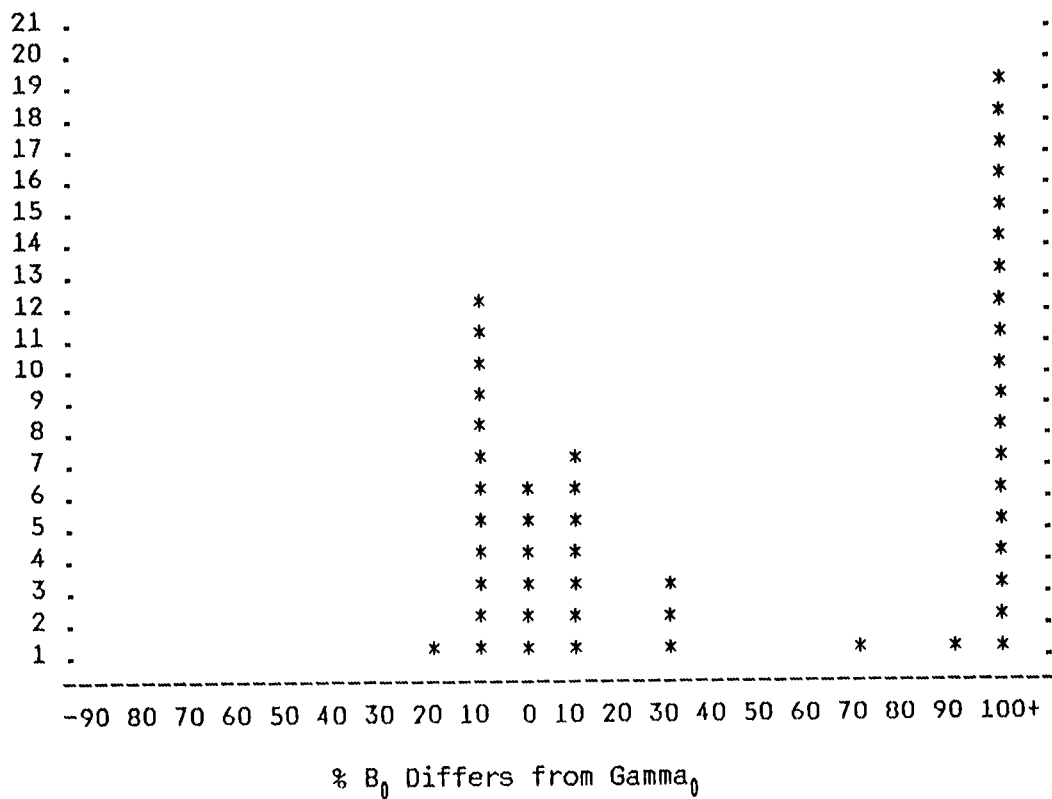


Chart of Estimates of B_0 with :

$\Gamma_0 = 50$ $\Phi = 0.001$
 $n = 45$

Source : 50out01
 Output File : 50out01.his

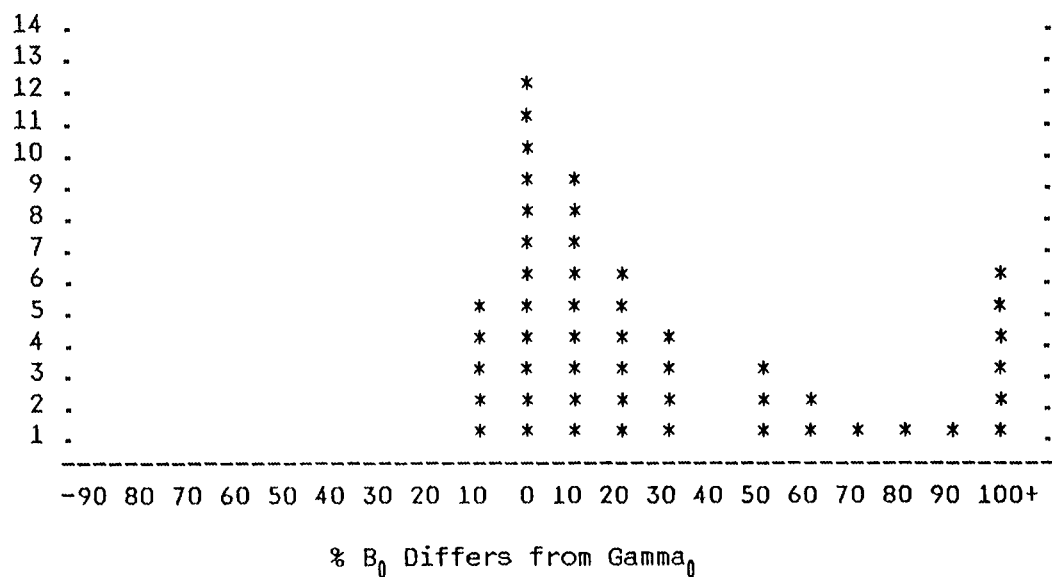


Chart of Estimates of B_0 with :

$\Gamma_0 = 100$ $\Phi = 0.001$
 $n = 10$

Source : 100out01
 Output File : 100out01.his

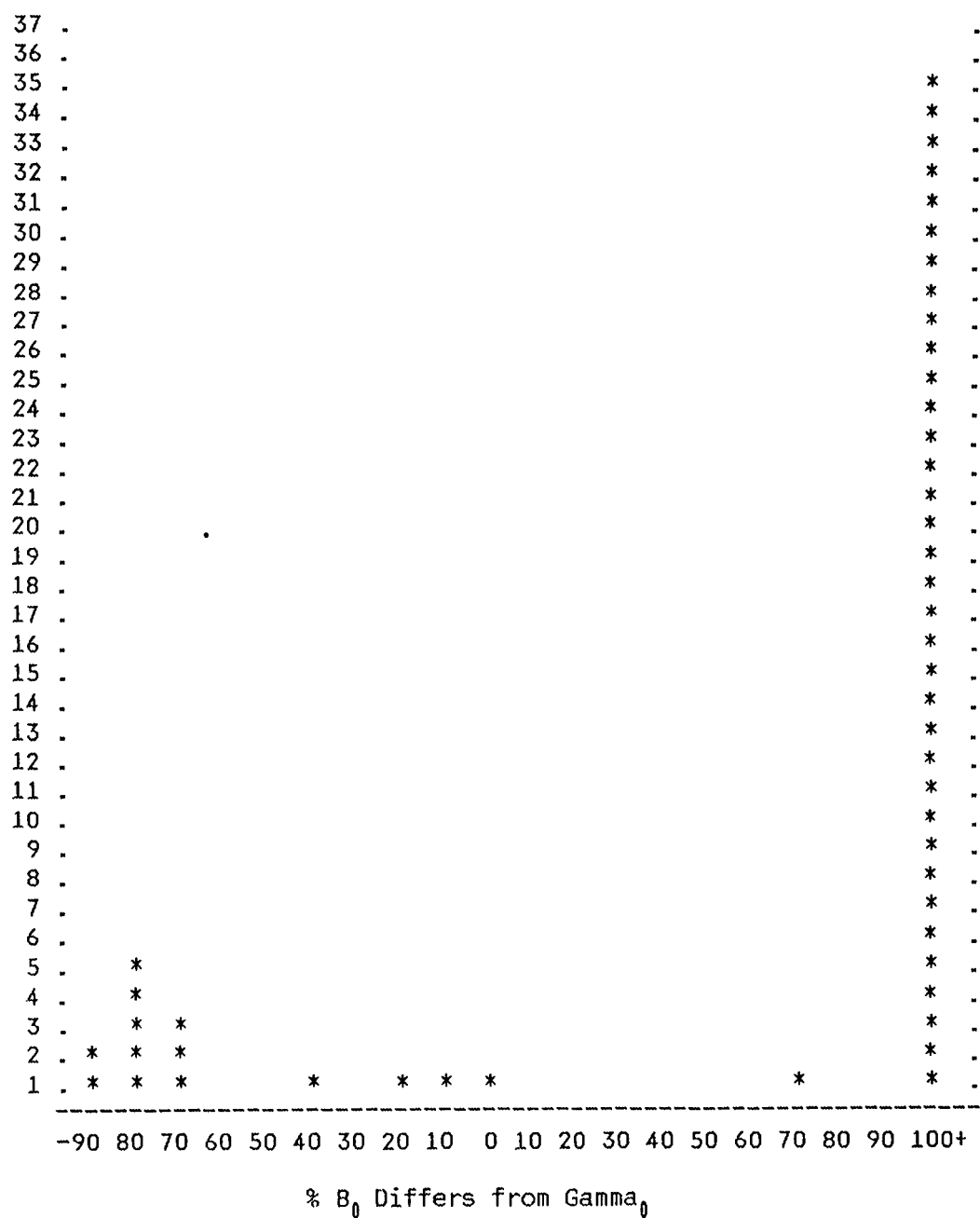


Chart of Estimates of B_0 with :

$\Gamma_0 = 100$ $\Phi = 0.001$
 $n = 20$

Source : 100out01
 Output File : 100out01.his

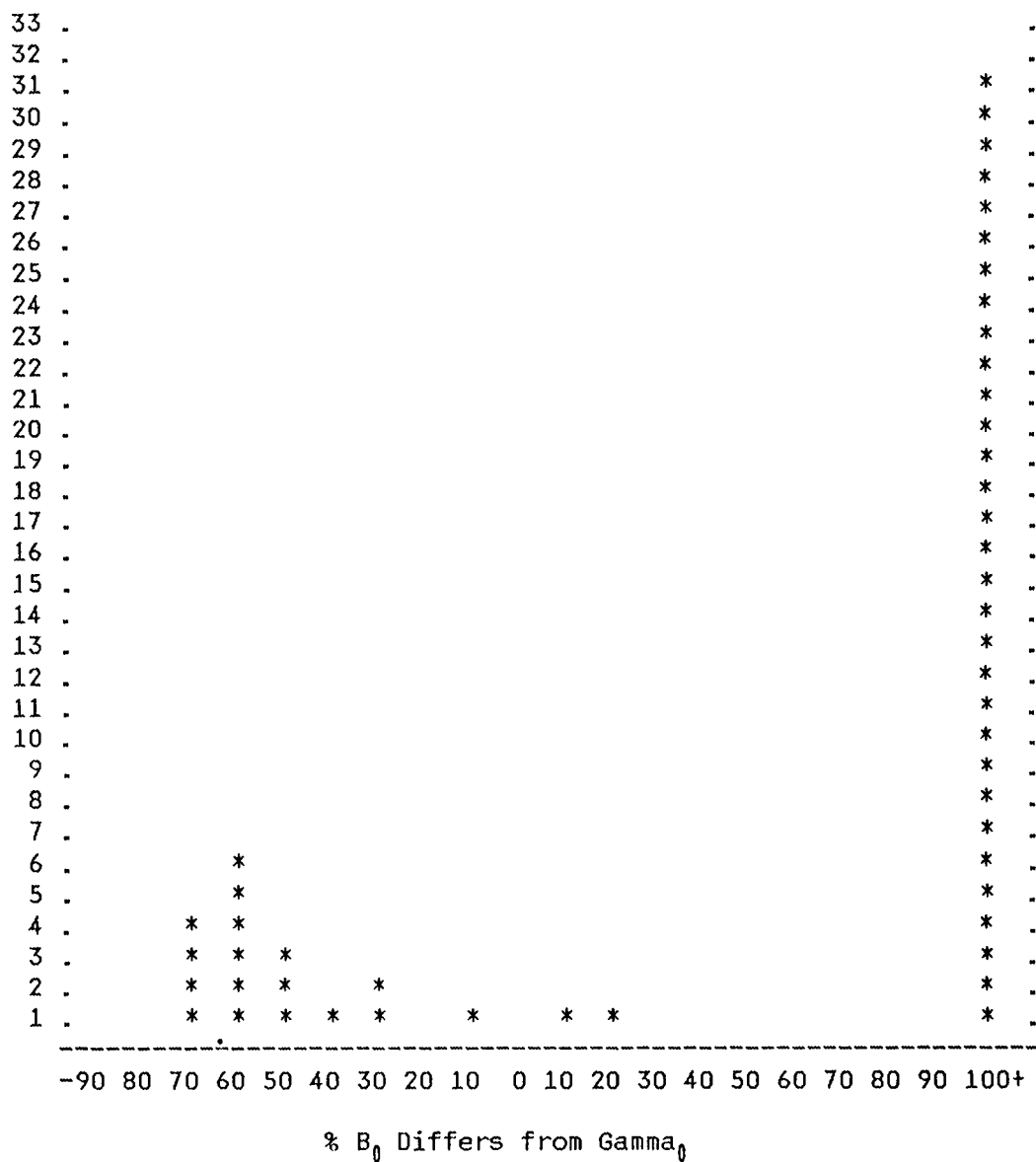


Chart of Estimates of B_0 with :

$\Gamma_0 = 100$ $\Phi = 0.001$
 $n = 30$

Source : 100out01
 Output File : 100out01.his

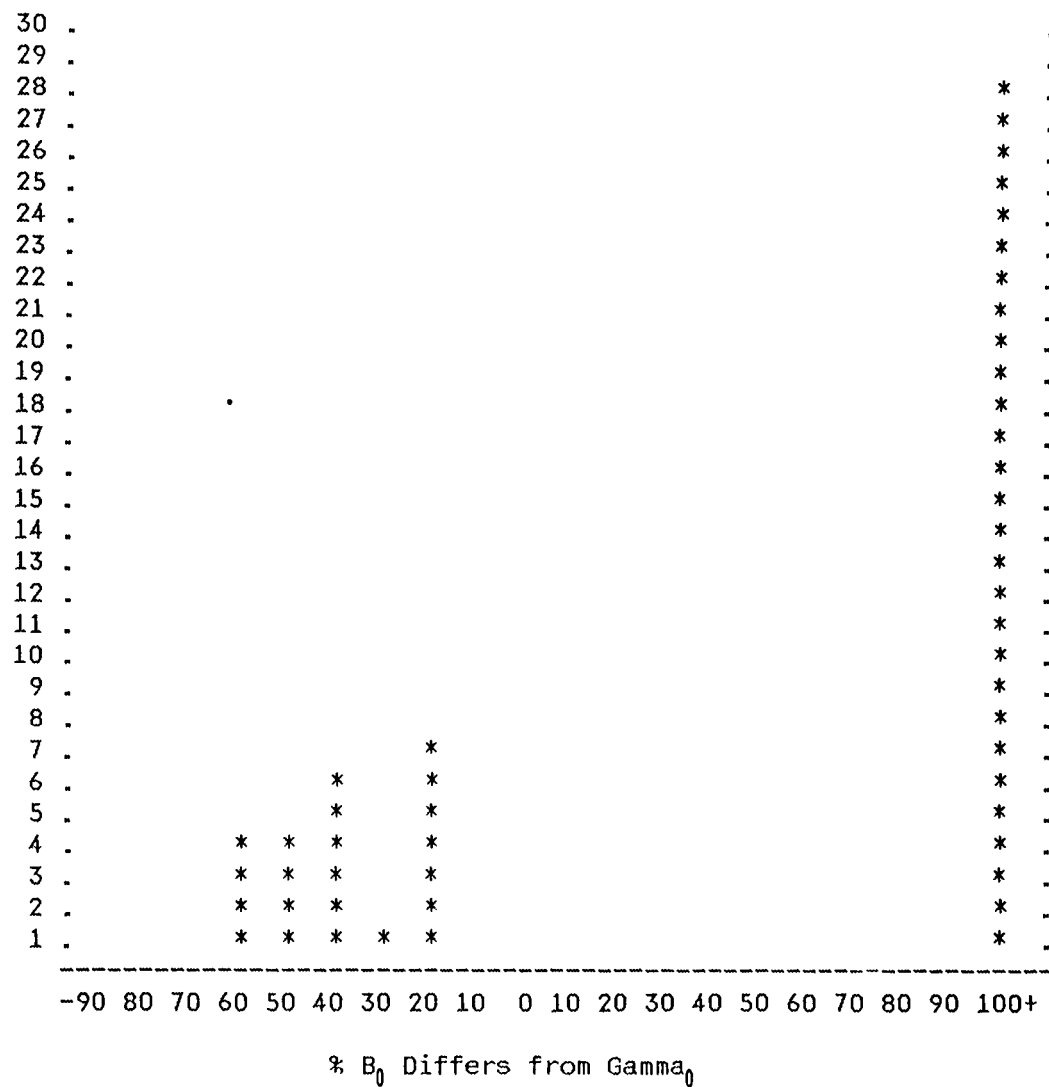


Chart of Estimates of B_0 with :

$\Gamma_0 = 100$ $\Phi = 0.001$
 $n = 40$

Source : 100out01
 Output File : 100out01.his

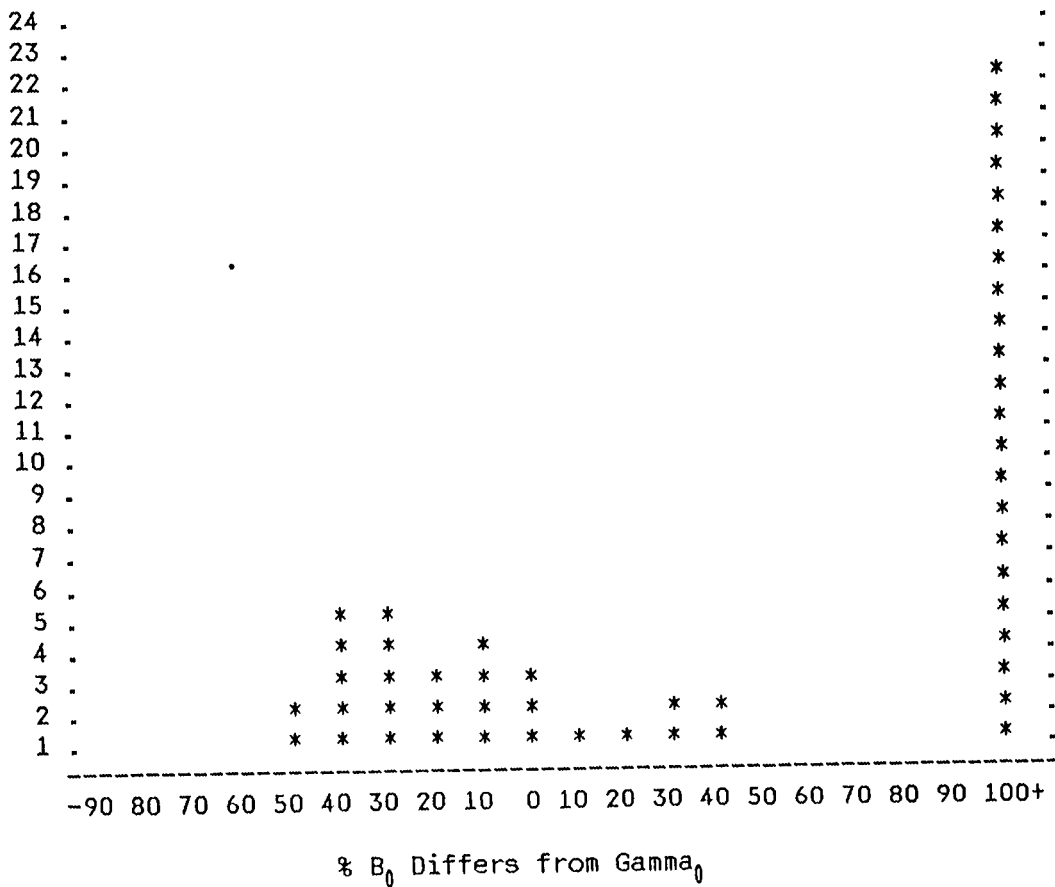


Chart of Estimates of B_0 with :

$\Gamma_0 = 100$ $\Phi = 0.001$
 $n = 50$

Source : 100out01
 Output File : 100out01.his

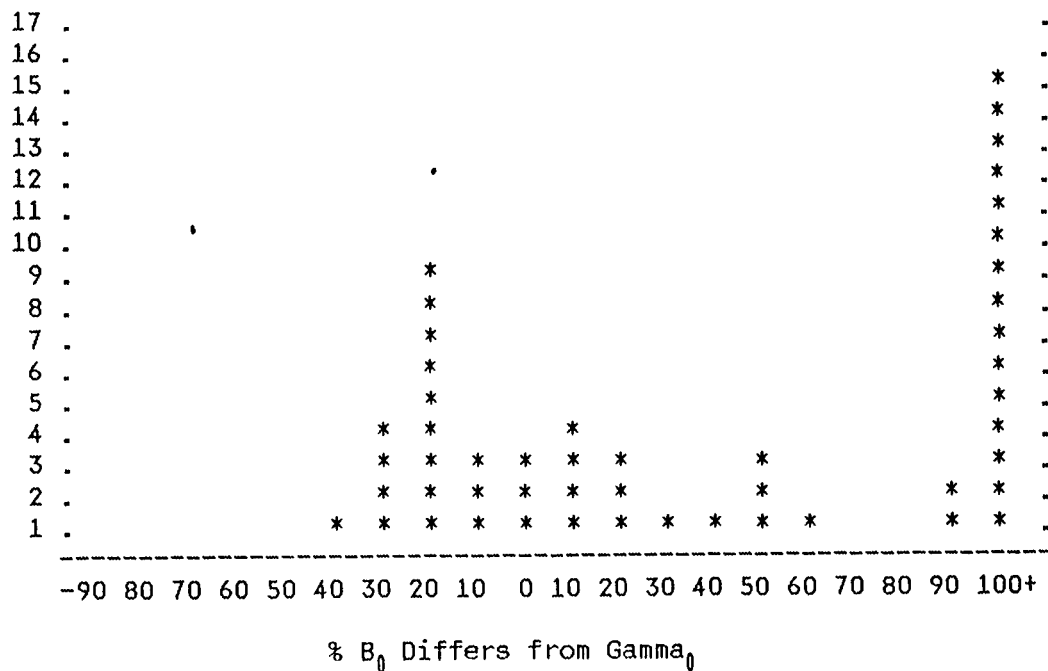


Chart of Estimates of B_0 with :

$\text{Gamma}_0 = 100$ $\text{Phi} = 0.001$
 $n = 60$

Source : 100out01
 Output File : 100out01.his

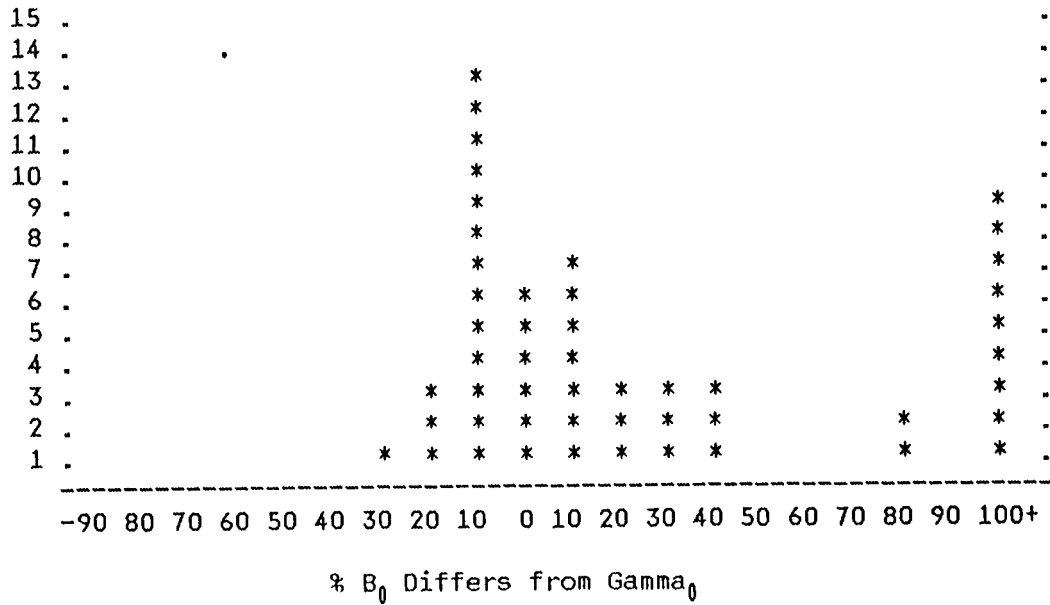


Chart of Estimates of B_0 with :

$\text{Gamma}_0 = 100$ $\text{Phi} = 0.001$
 $n = 70$

Source : 100out01
 Output File : 100out01.his

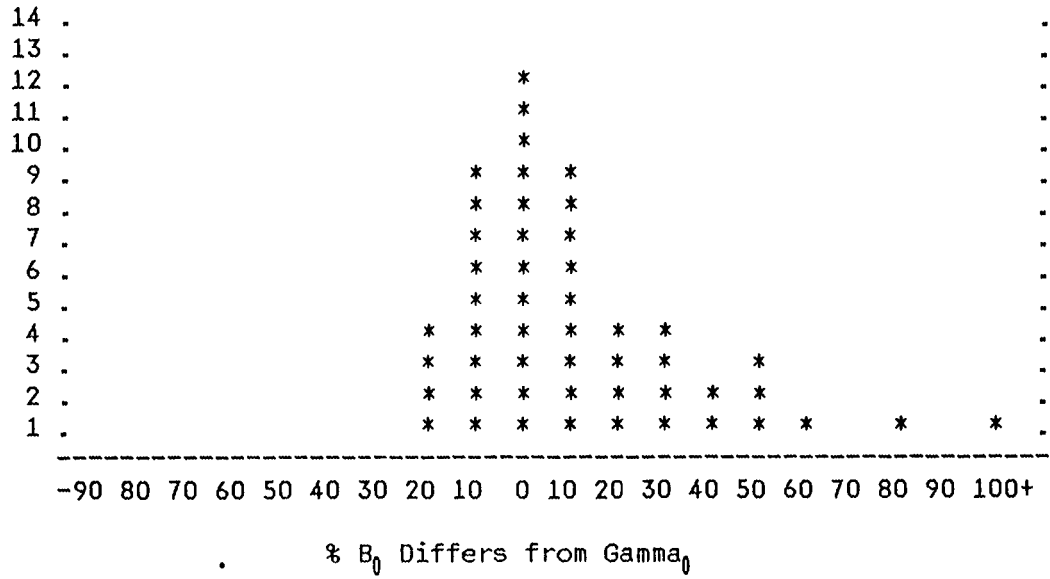


Chart of Estimates of B_0 with :

$\Gamma_0 = 100$ $\Phi = 0.001$
 $n = 80$

Source : 100out01
 Output File : 100out01.his

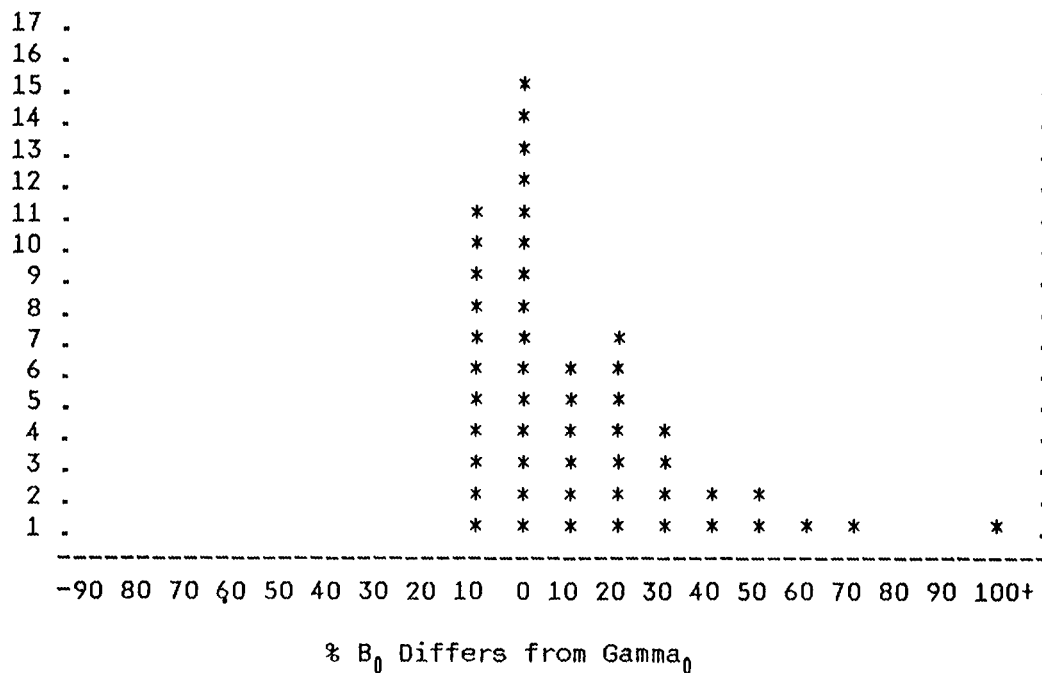


Chart of Estimates of B_0 with :

Gamma₀ = 100 Phi = 0.001
 n = 90

Source : 100out01
 Output File : 100out01.his

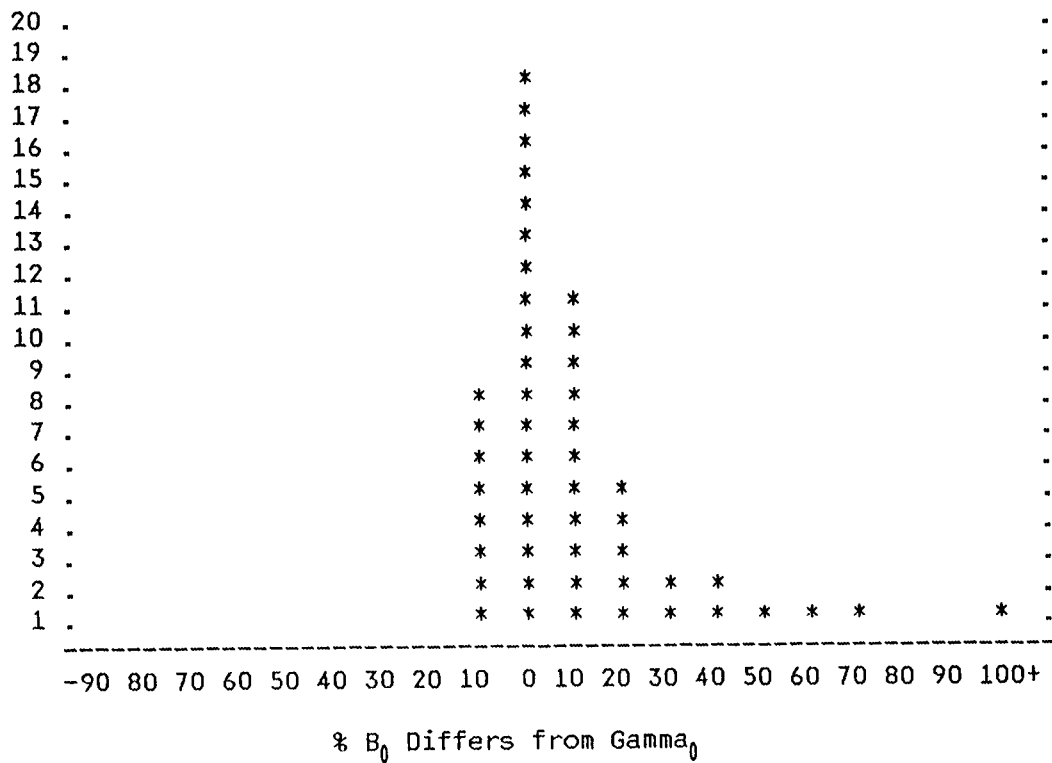


Chart of Estimates of B_0 with :

Gamma₀ = 200 Phi = 0.001
 n = 20

Source : 200out01
 Output File : 200out01.his

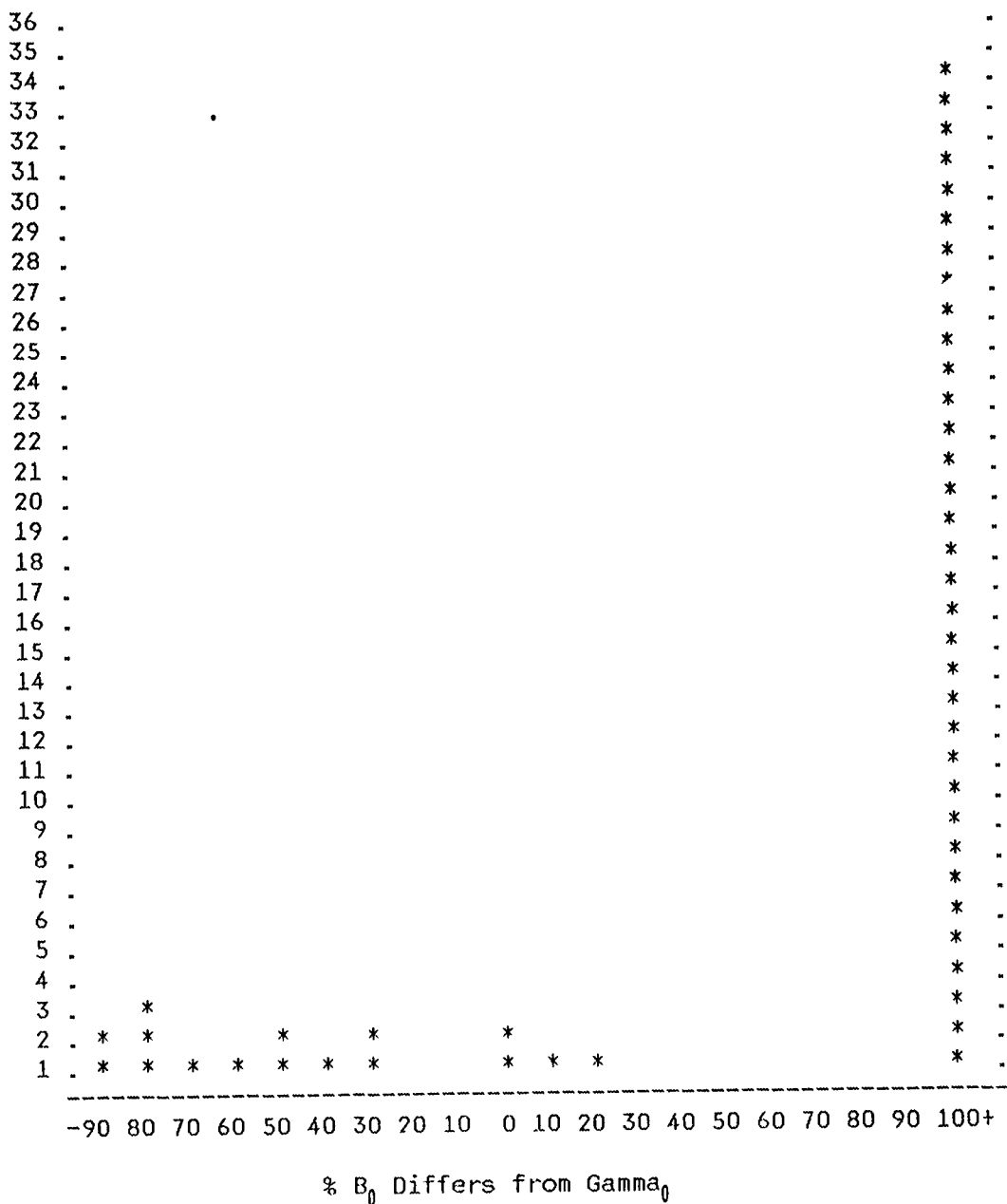


Chart of Estimates of B_0 with :

$\Gamma_0 = 200$ $\Phi = 0.001$
 $n = 40$

Source : 200out01
 Output File : 200out01.his

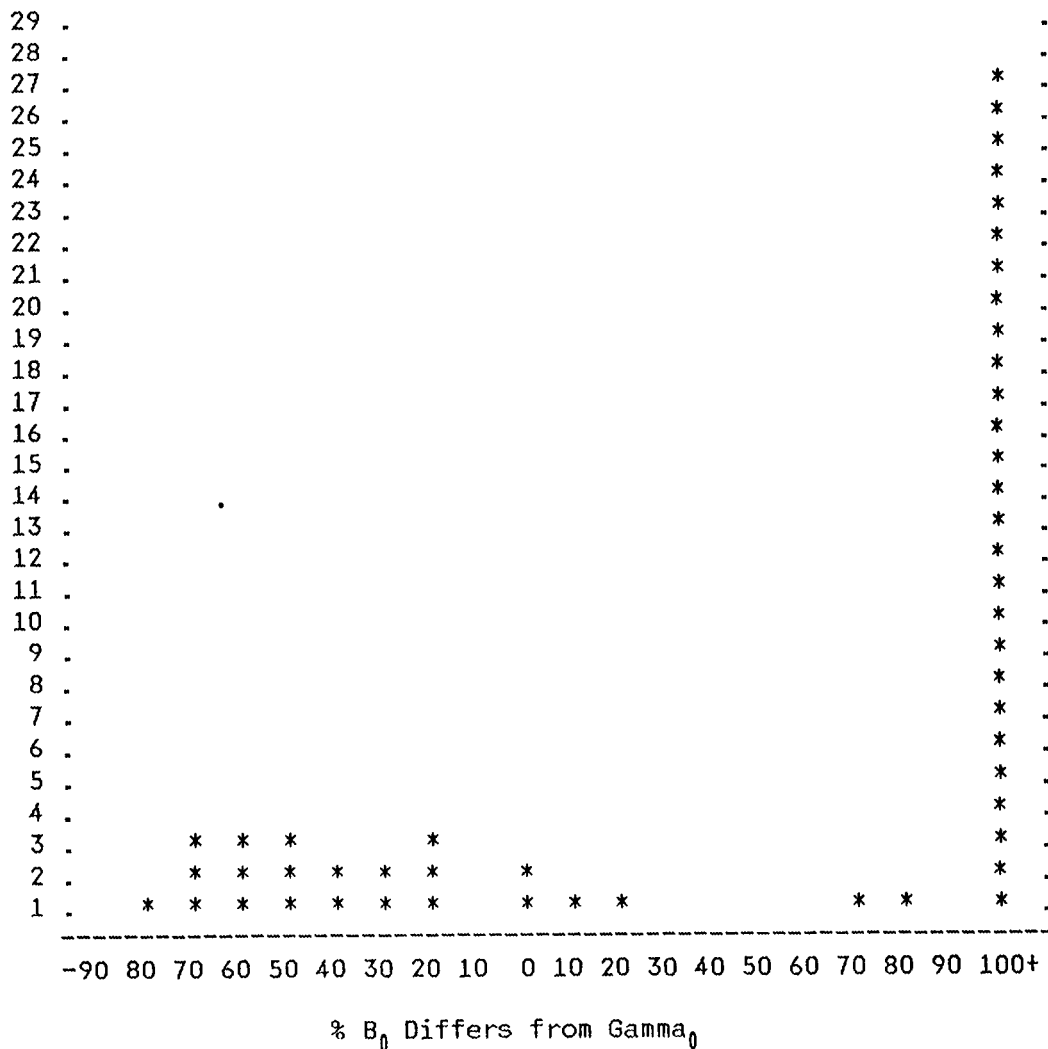


Chart of Estimates of B_0 with :

$\Gamma_0 = 200$ $\Phi = 0.001$
 $n = 60$

Source : 200out01
 Output File : 200out01.his

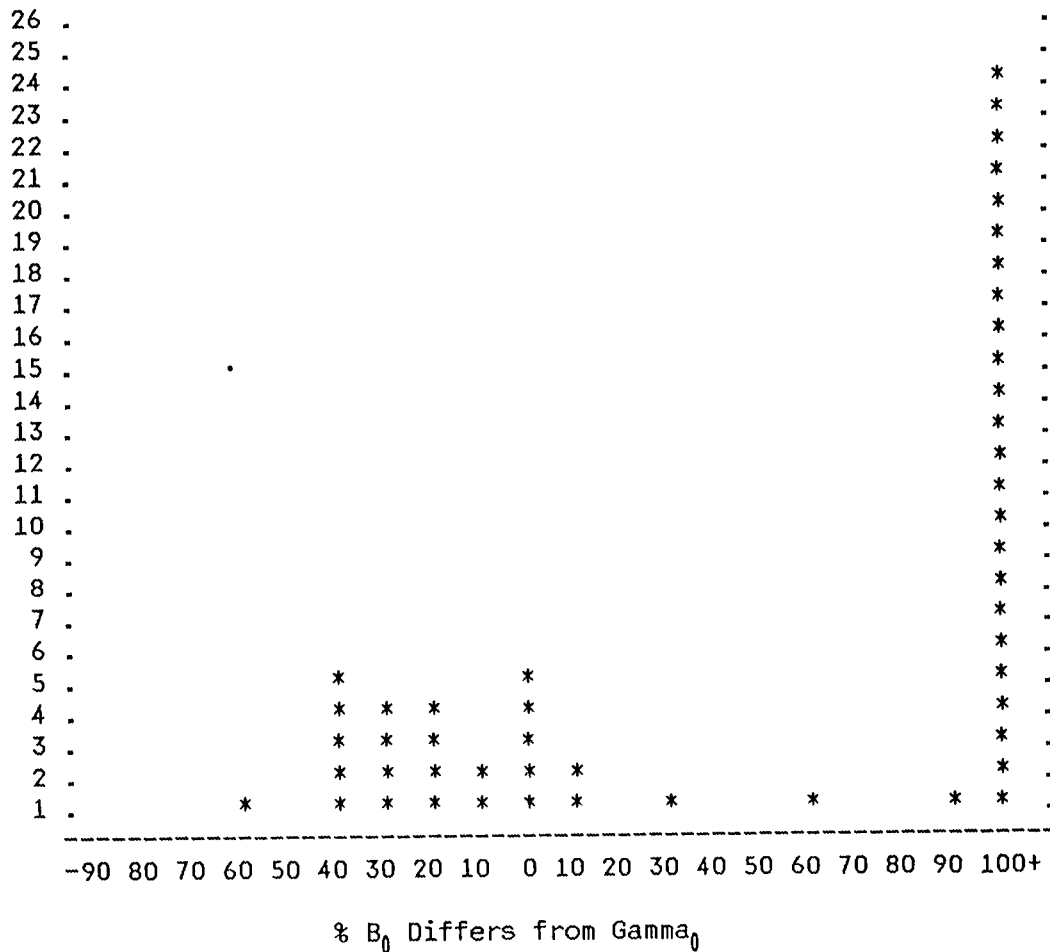


Chart of Estimates of B_0 with :

$\Gamma_0 = 200$ $\Phi = 0.001$
 $n = 80$

Source : 200out01
 Output File : 200out01.his

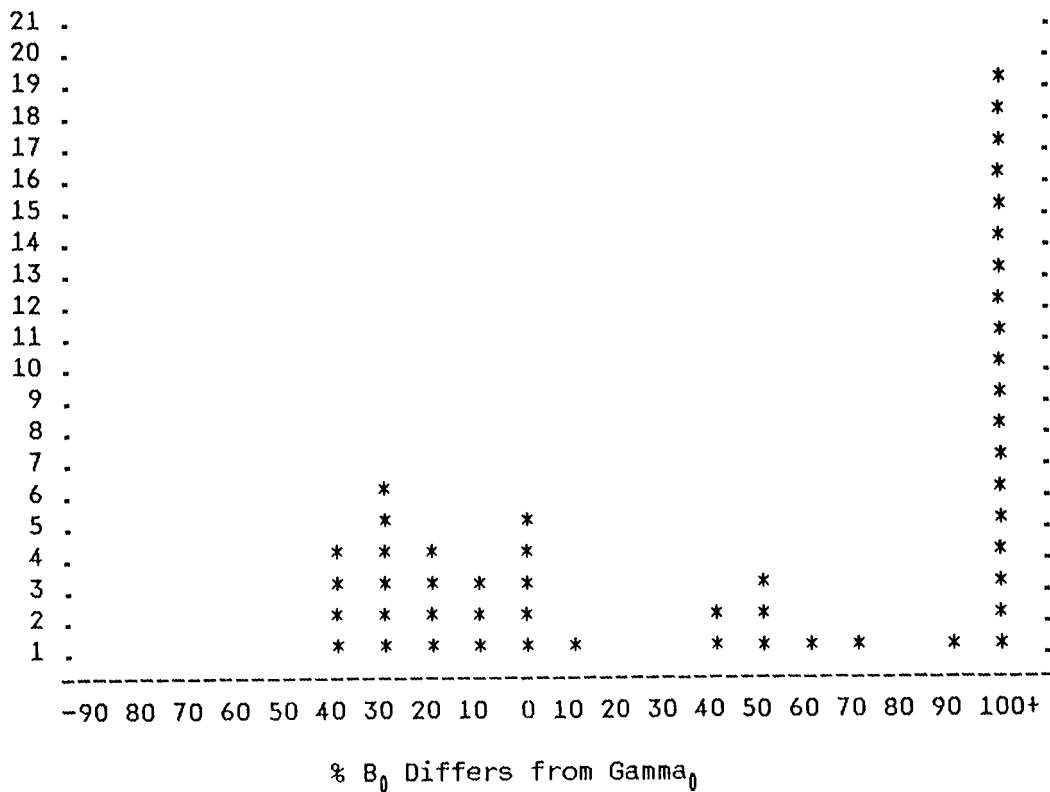


Chart of Estimates of B_0 with :

$\Gamma_0 = 200$ $\Phi = 0.001$
 $n = 100$

Source : 200out01
 Output File : 200out01.his

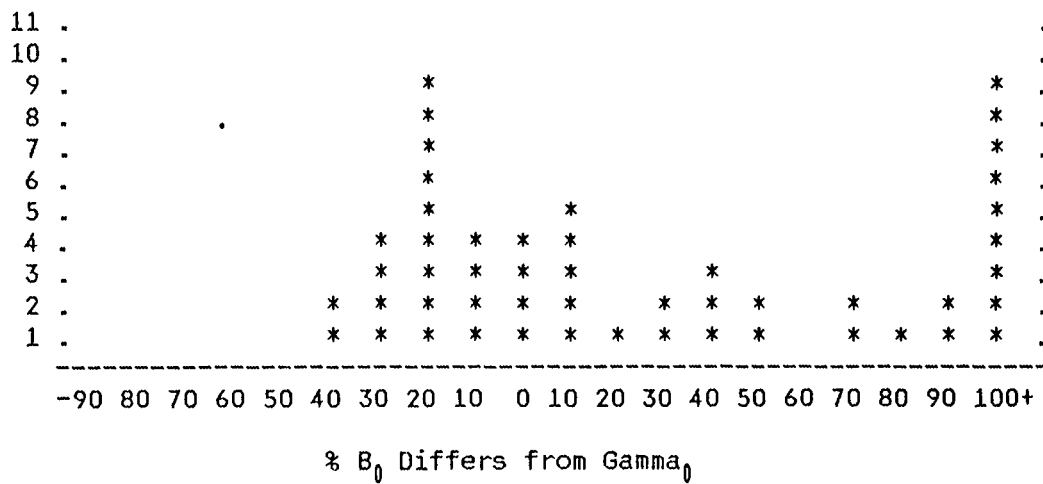


Chart of Estimates of B_0 with :

$\Gamma_0 = 200$ $\Phi = 0.001$
 $n = 120$

Source : 200out01
 Output File : 200out01.his

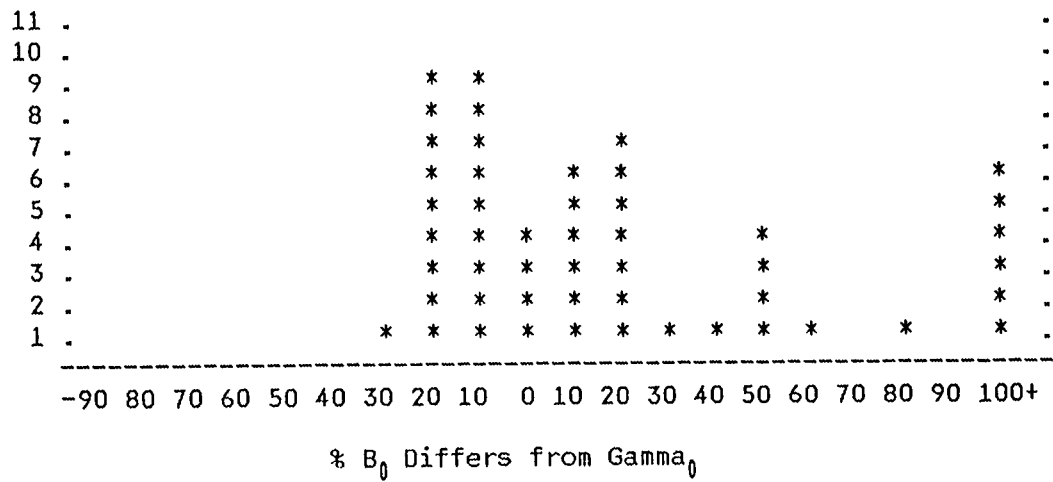


Chart of Estimates of B_0 with :

$\text{Gamma}_0 = 200$ $\Phi = 0.001$
 $n = 140$

Source : 200out01
 Output File : 200out01.his

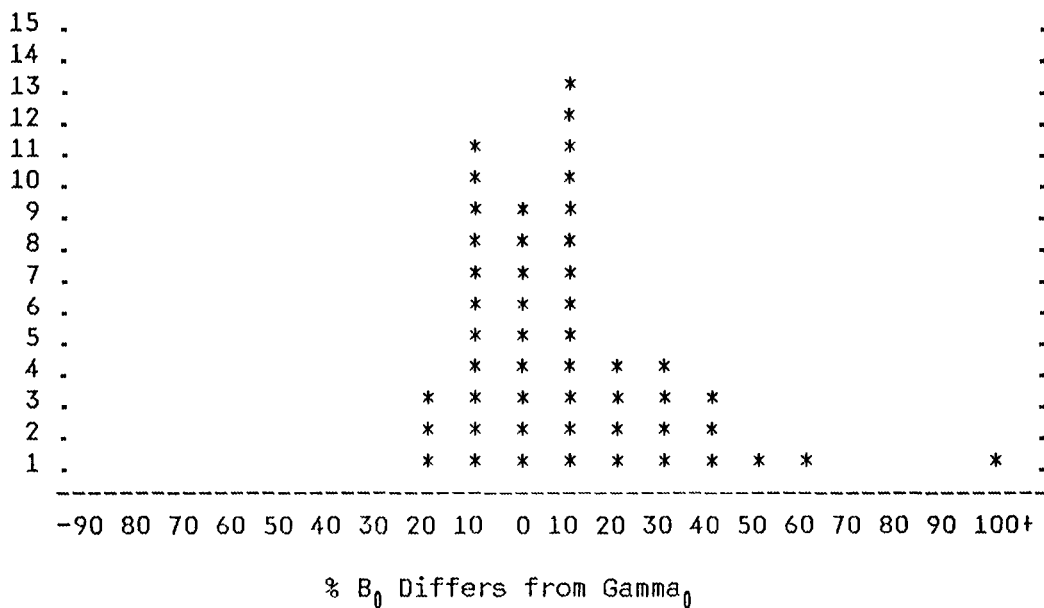


Chart of Estimates of B_0 with :

$\Gamma_0 = 200$, $\Phi = 0.001$
 $n = 160$

Source : 200out01
 Output File : 200out01.his

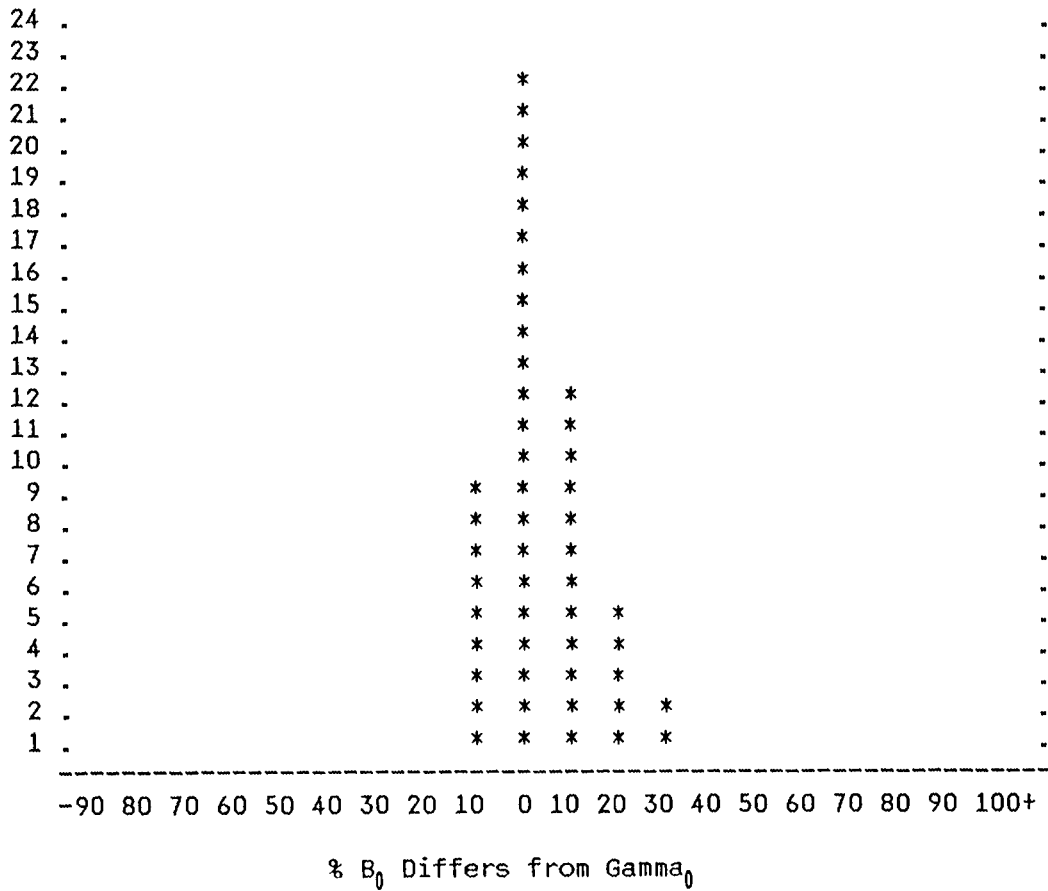
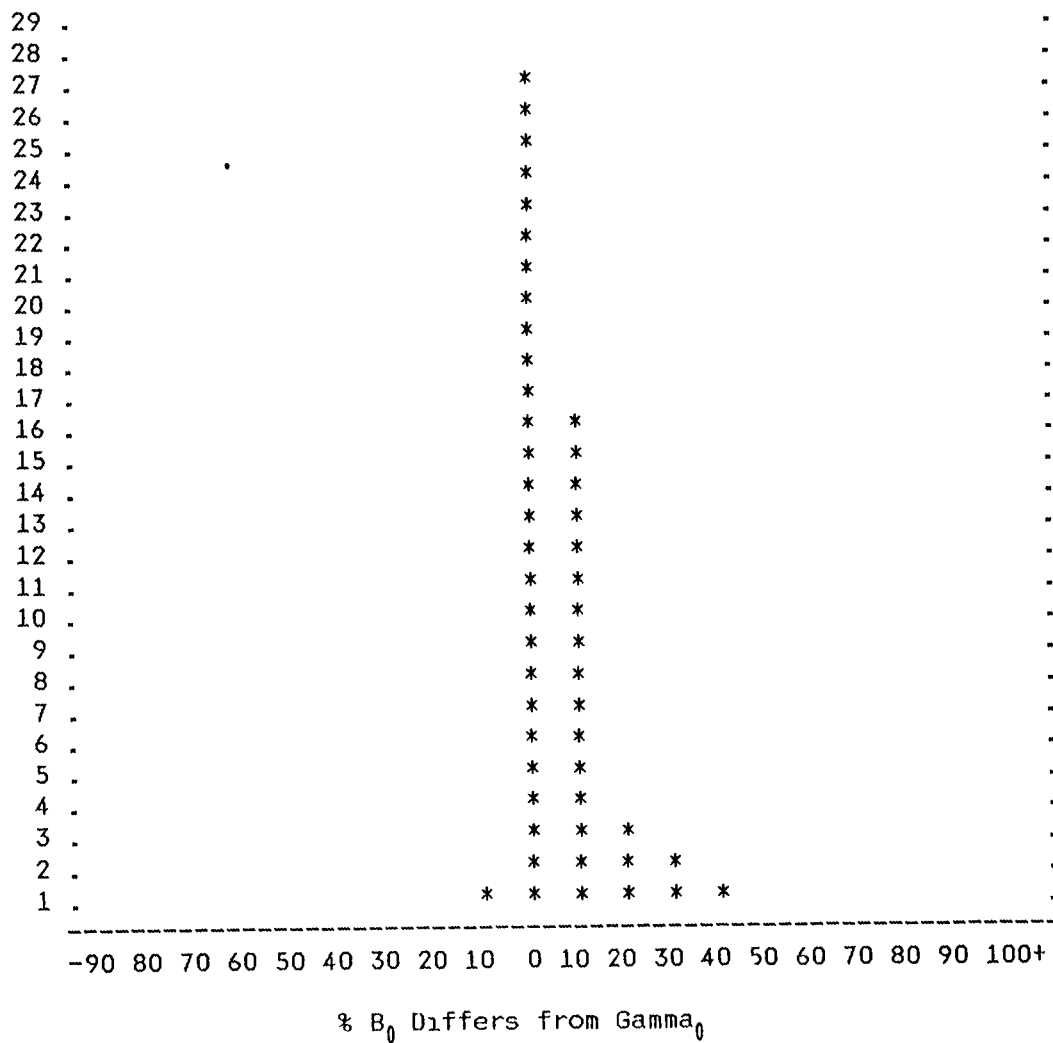


Chart of Estimates of B_0 with :

$\Gamma_0 = 200$ $\Phi = 0.001$
 $n = 180$

Source : 200out01
 Output File : 200out01.his



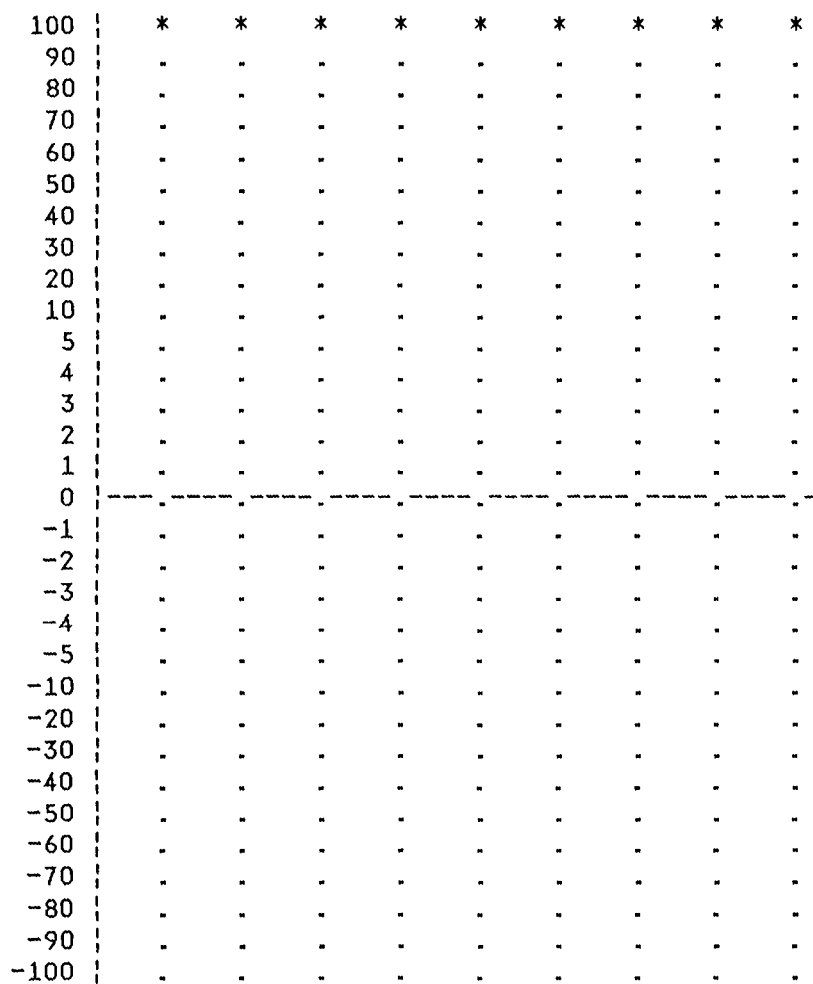
Summary Graph of % Mean B_0 Estimate differs from Γ_{γ_0}

Gamma_0 = 50 Phi = 0.001

Source : 50out01

Output : 50out01.gph

%off



Failures Generated

Me	Mean	B_1	Std	Mean	B_0	Std	tsum	Mean	te
5	0.00136750	0.00367522	1248.42	1488.88	304.5	105.9			
10	0.00147675	0.00279149	884.04	829.23	1207.7	227.1			
15	0.00100312	0.00165105	1453.48	1894.83	2763.1	359.8			
20	0.00087137	0.00109232	1168.41	1885.41	4986.2	514.7			
25	0.00088919	0.00105804	874.91	1593.78	8082.6	695.2			
30	0.00081481	0.00076650	776.62	1784.15	12200.0	914.6			
35	0.00075247	0.00057464	757.89	2031.99	17490.5	1174.6			
40	0.00070886	0.00045880	294.65	934.83	24372.6	1546.5			
45	0.00075474	0.00031450	349.03	1699.32	34147.7	2366.4			

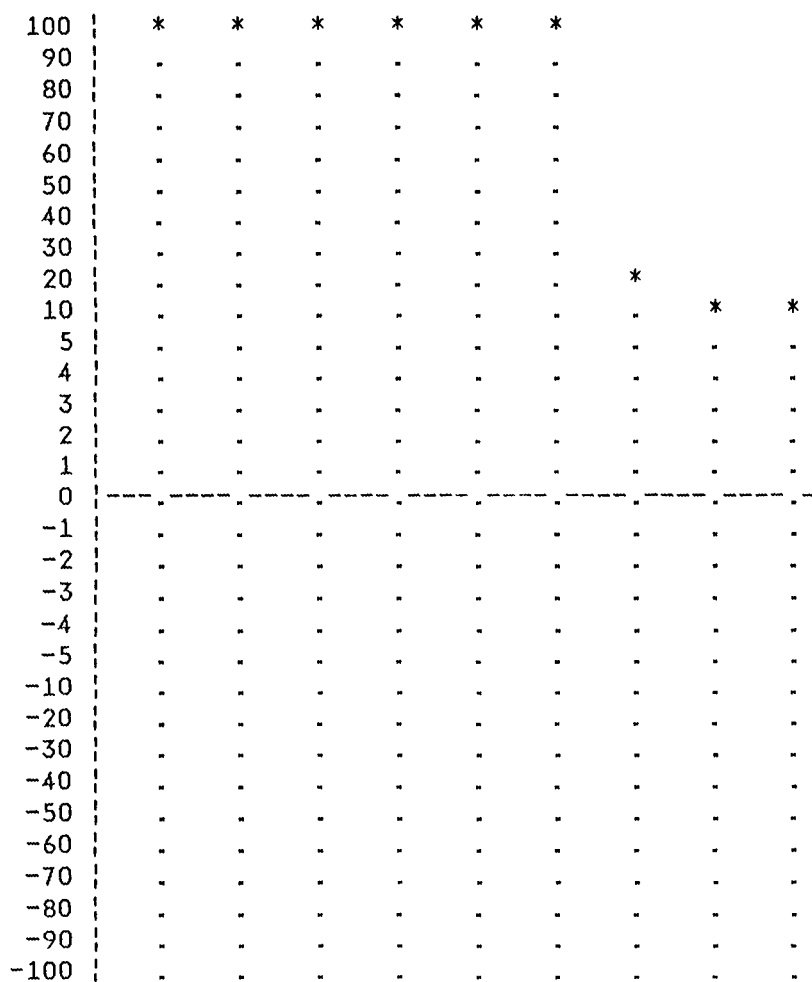
Summary Graph of % Mean B_0 Estimate differs from Γ_{γ_0}

$\Gamma_{\gamma_0} = 100$ $\Phi_{\gamma_0} = 0.001$

Source : 100out01

Output : 100out01.gph

%off



1 2 3 4 5 6 7 8 9 # Failures Generated
0 0 0 0 0 0 0 0 0

Me	B_1		B_0		tsum	
	Mean	Std	Mean	Std	Mean	te Mean
10	0.00191850	0.00379749	1591.31	1327.52	557.6	102.9
20	0.00137475	0.00226914	1784.37	1646.17	2216.1	218.0
30	0.00117525	0.00159097	2846.65	3694.71	5069.6	344.0
40	0.00093056	0.00100241	2846.53	4418.66	9329.6	492.9
50	0.00086156	0.00062904	987.87	2807.85	15242.3	674.8
60	0.00086331	0.00047275	758.97	2445.93	23107.9	885.4
70	0.00096375	0.00034101	117.09	52.26	33483.8	1175.2
80	0.00093484	0.00025343	112.07	23.46	47245.8	1564.0
90	0.00091808	0.00021738	111.31	20.98	66033.2	2216.5

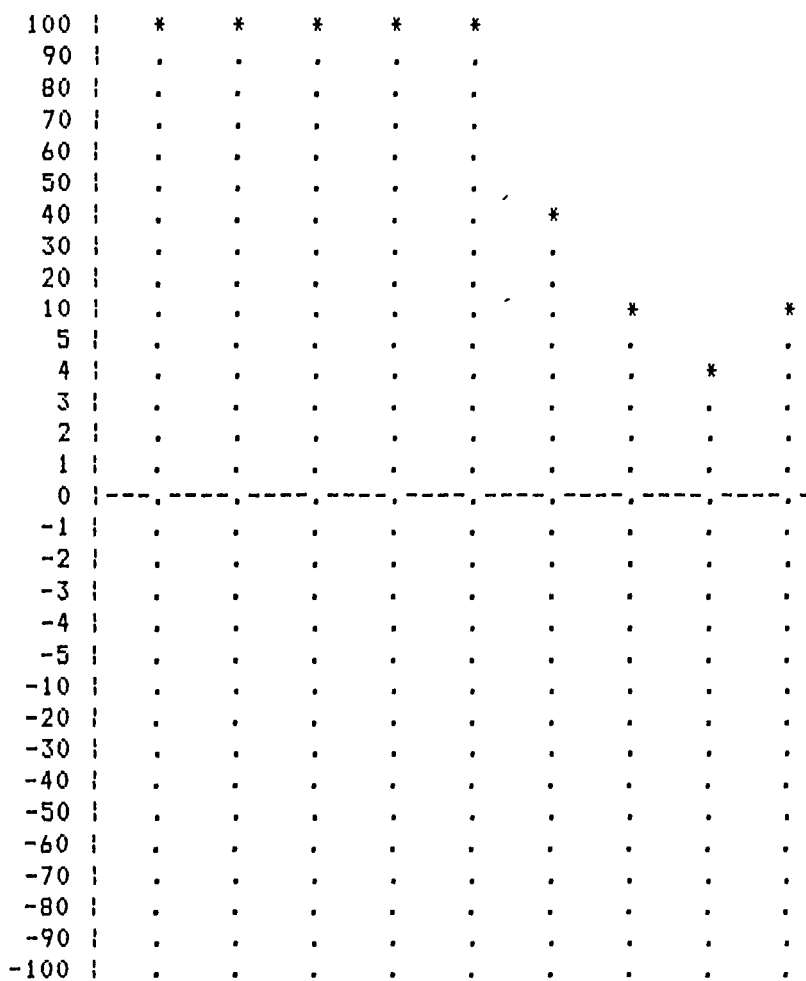
Summary Graph of % Mean B_0 Estimate differs from Γ_{γ_0}

$\Gamma_{\gamma_0} = 200$ $\Phi_1 = 0.001$

Source : 200out01

Output : 200out01.gph

%off



2 4 6 8 1 1 1 1 1
0 0 0 0 0 0 0 0 0 # Failures Generated

Me	B_1		B_0		t_{sum}	t_e
	Mean	Std	Mean	Std	Mean	Mean
20	0.00139100	0.00297119	3209.80	2438.08	1119.1	105.4
40	0.00109850	0.00153999	5692.62	7112.14	4411.5	219.6
60	0.00077250	0.00079156	6097.46	10594.21	10159.8	348.5
80	0.00080875	0.00067817	3285.72	6909.38	18675.1	498.5
100	0.00091416	0.00054024	2645.41	9549.14	30494.9	681.8
120	0.00094406	0.00040844	272.89	222.85	46400.6	903.9
140	0.00094550	0.00024369	221.66	50.05	67287.9	1184.3
160	0.00097852	0.00015416	207.87	19.91	94830.3	1583.9
180	0.00094485	0.00012528	210.78	19.99	132685.0	2217.5

Appendix B

This appendix contains two types of graphs showing the modeling results for thirty replications. The first graphs are scatter plot of trials for the different total fault counts used and number of failures generated with 30 replications. Each scatter plot graph shows how much each of the 50 generated estimates for B_0 , total program faults, differ (within plus or minus 5%) from the actual parameter Γ_0 . n indicates the number of failures times produced, r indicates the number of replications (30), and ϕ is the per fault failure rate used.

The second graphs are summary graphs of the scatter plot graphs. Each summary graph shows how much the mean of the 50 calculated estimates for B_0 differs from the actual parameter Γ_0 for the different number of failures generated using 30 repetitions.

APPENDIX B INDEX

B1-1	Scatter Plots for 50 total faults
B2-1	Scatter Plots for 100 total faults
B3-1	Scatter Plots for 200 total faults
B4-1	Summary Graph for 50 total faults
B5-1	Summary Graph for 100 total faults
B6-1	Summary Graph for 200 total faults

Chart of Estimates of B_0 with :

Gamma₀ = 50 Phi = 0.001
n = 5 r = 30

Source : 50out3
Output File : 50out3.his

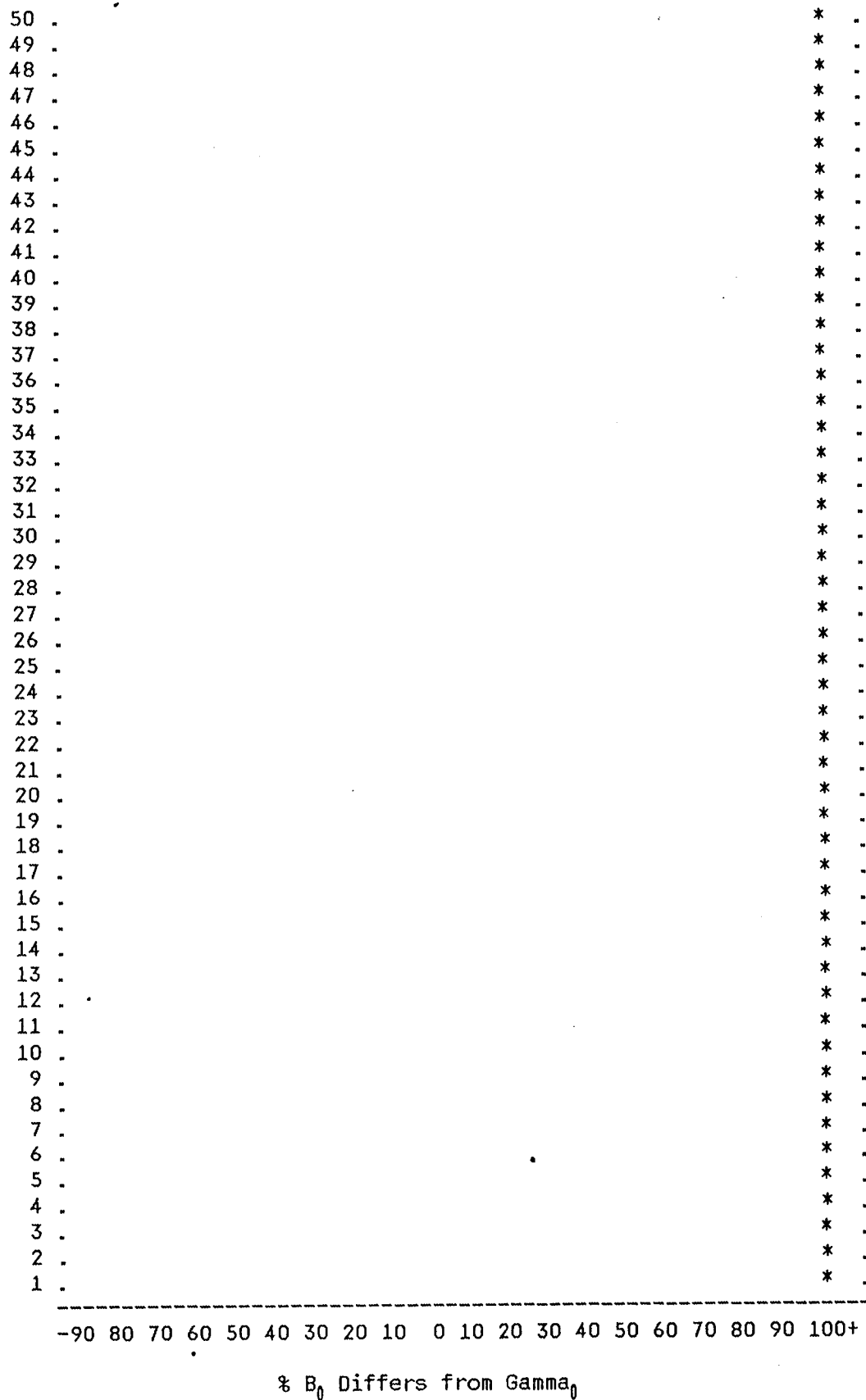
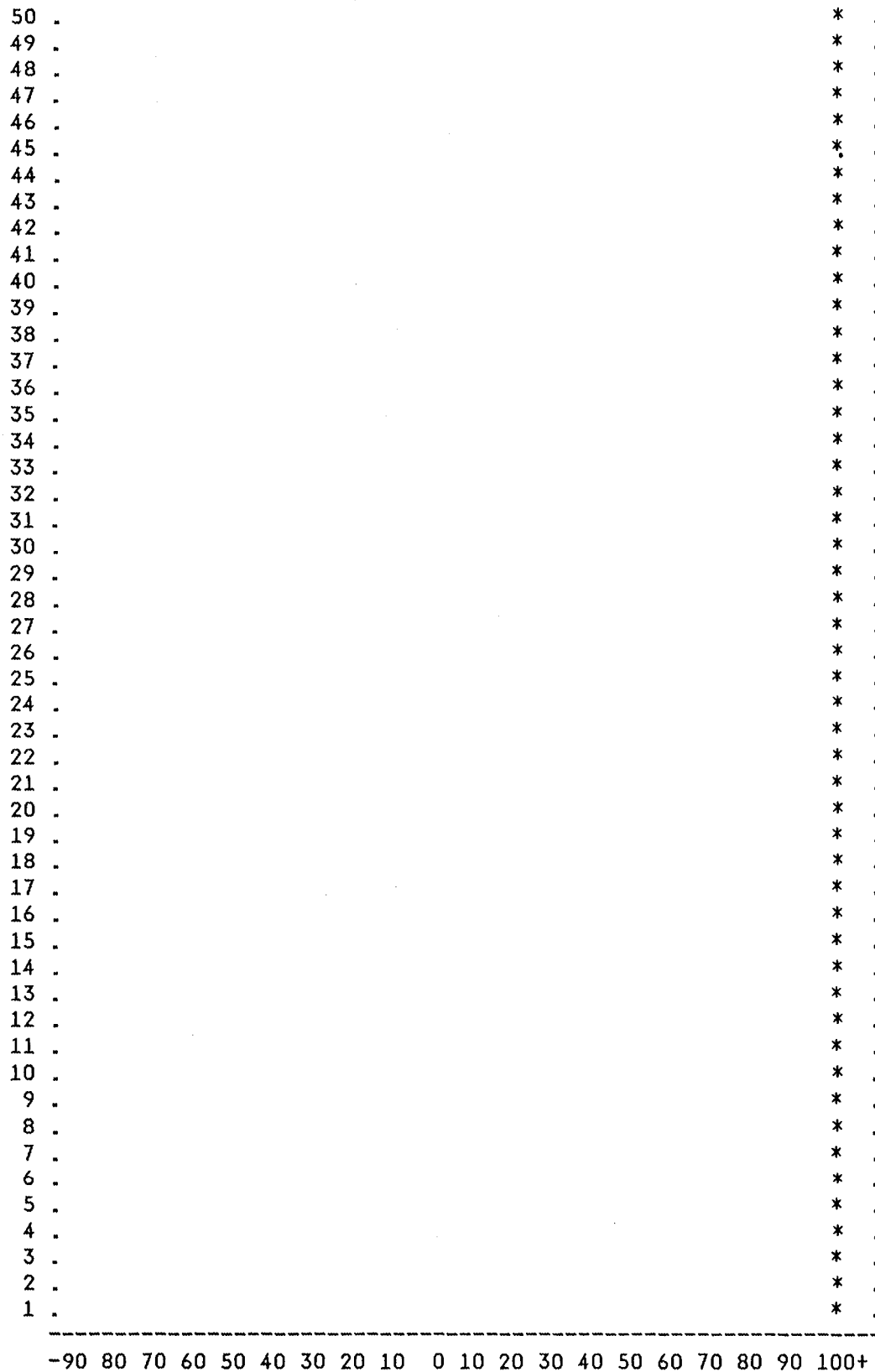


Chart of Estimates of B_0 with :

Gamma₀ = 50 Phi = 0.001
 n = 10 r = 30

Source : 50out3
 Output File : 50out3.his

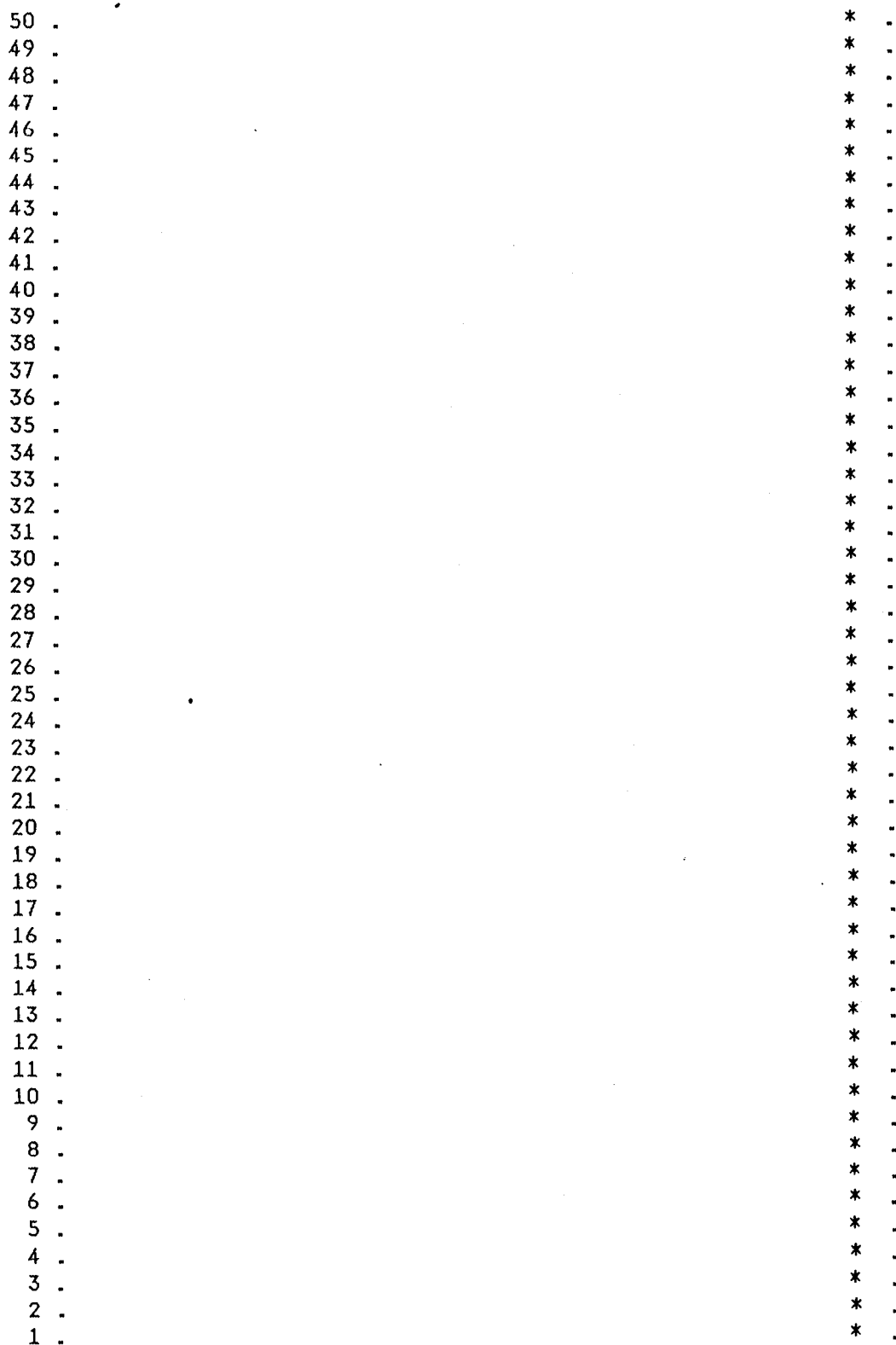


% B₀ Differs from Gamma₀

Chart of Estimates of B_0 with :

Gamma₀ = 50 Phi = 0.001
n = 15 r = 30

Source : 50out3
Output File : 50out3.his



-90 80 70 60 50 40 30 20 10 0 10 20 30 40 50 60 70 80 90 100+

% B_0 Differs from Gamma₀

Chart of Estimates of B_0 with :

Gamma₀ = 50 Phi = 0.001
n_r = 20 r = 30

Source : 50out3
Output File : 50out3.his

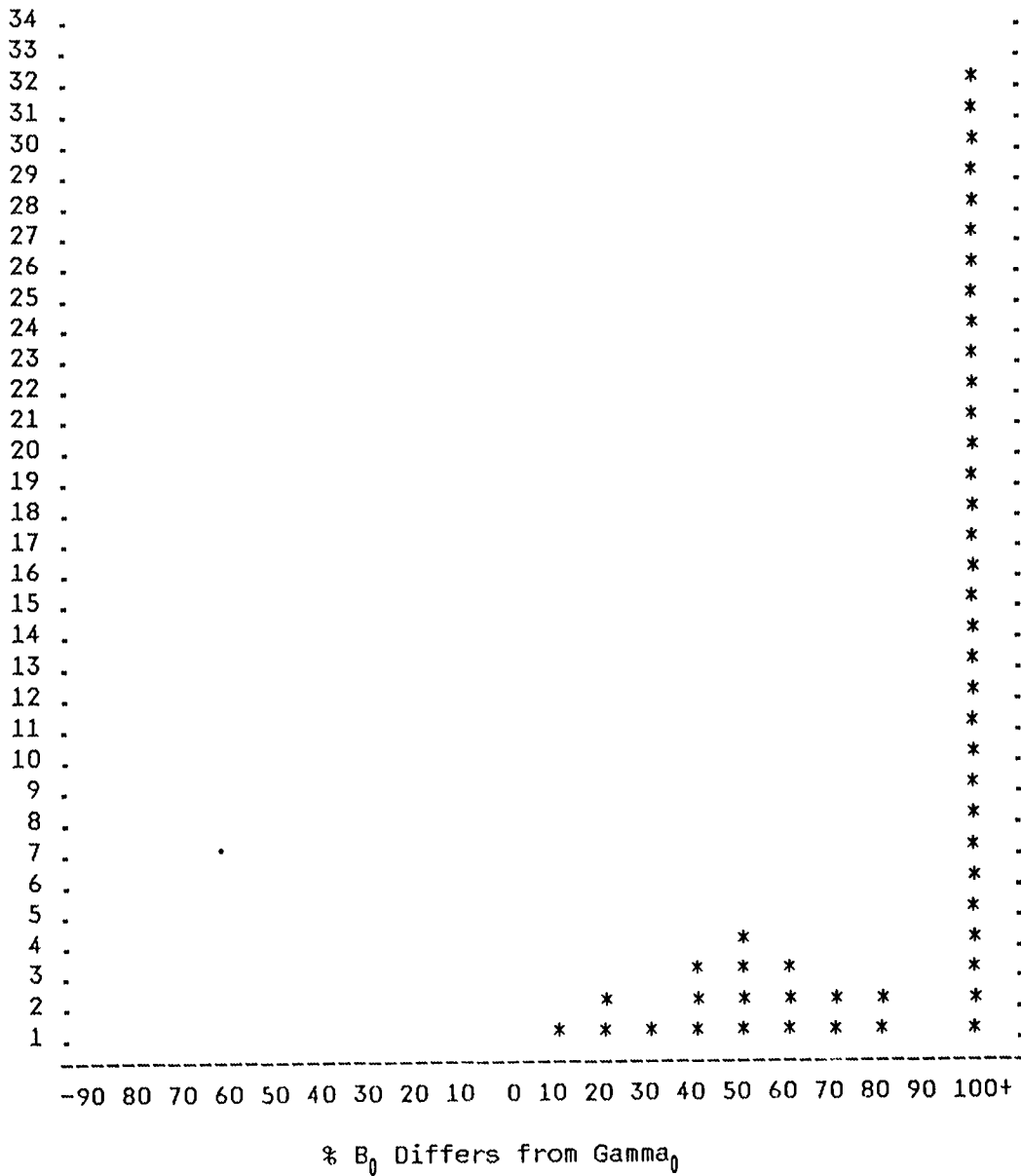


Chart of Estimates of B_0 with :

$\Gamma_0 = 50$ $\Phi = 0.001$
 $n = 25$ $r = 30$

Source : 50out3
 Output File : 50out3.his

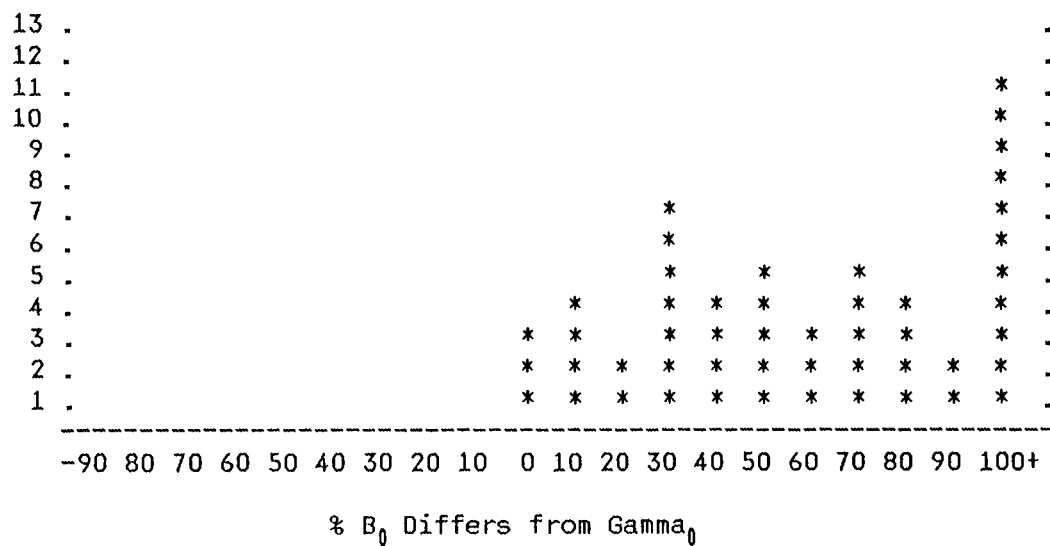


Chart of Estimates of B_0 with :

$\Gamma_0 = 50$ $\Phi = 0.001$
 $n = 30$ $r = 30$

Source : 50out3
 Output File : 50out3.his

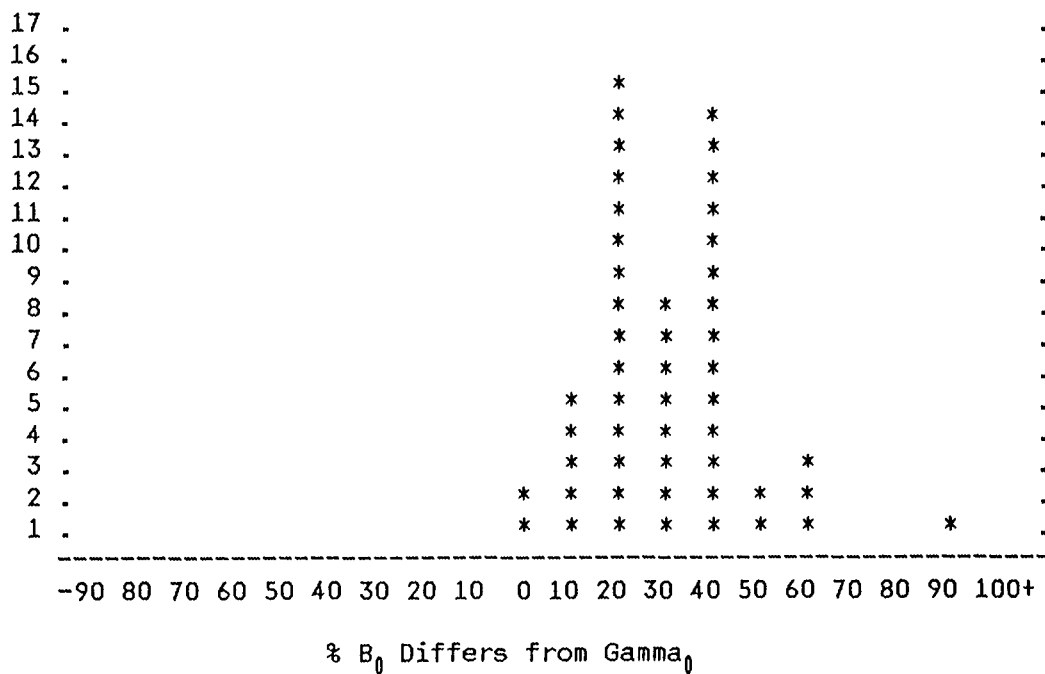


Chart of Estimates of B_0 with :

$\Gamma_0 = 50$ $\Phi = 0.001$
 $n = 35$ $r = 30$

Source : 50out3
 Output File : 50out3.his

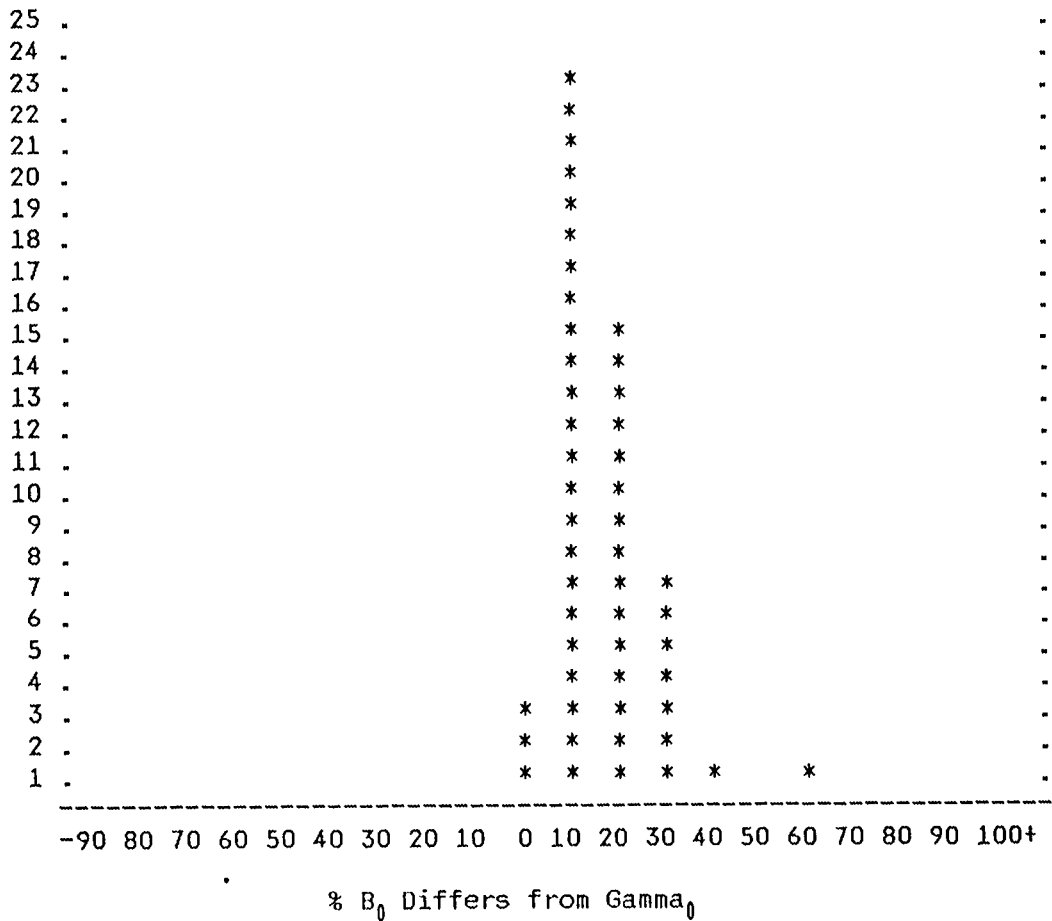


Chart of Estimates of B_0 with :

$\Gamma_0 = 50$ $\Phi = 0.001$
 $n = 40$ $r = 30$

Source : 50out3
 Output File : 50out3.his

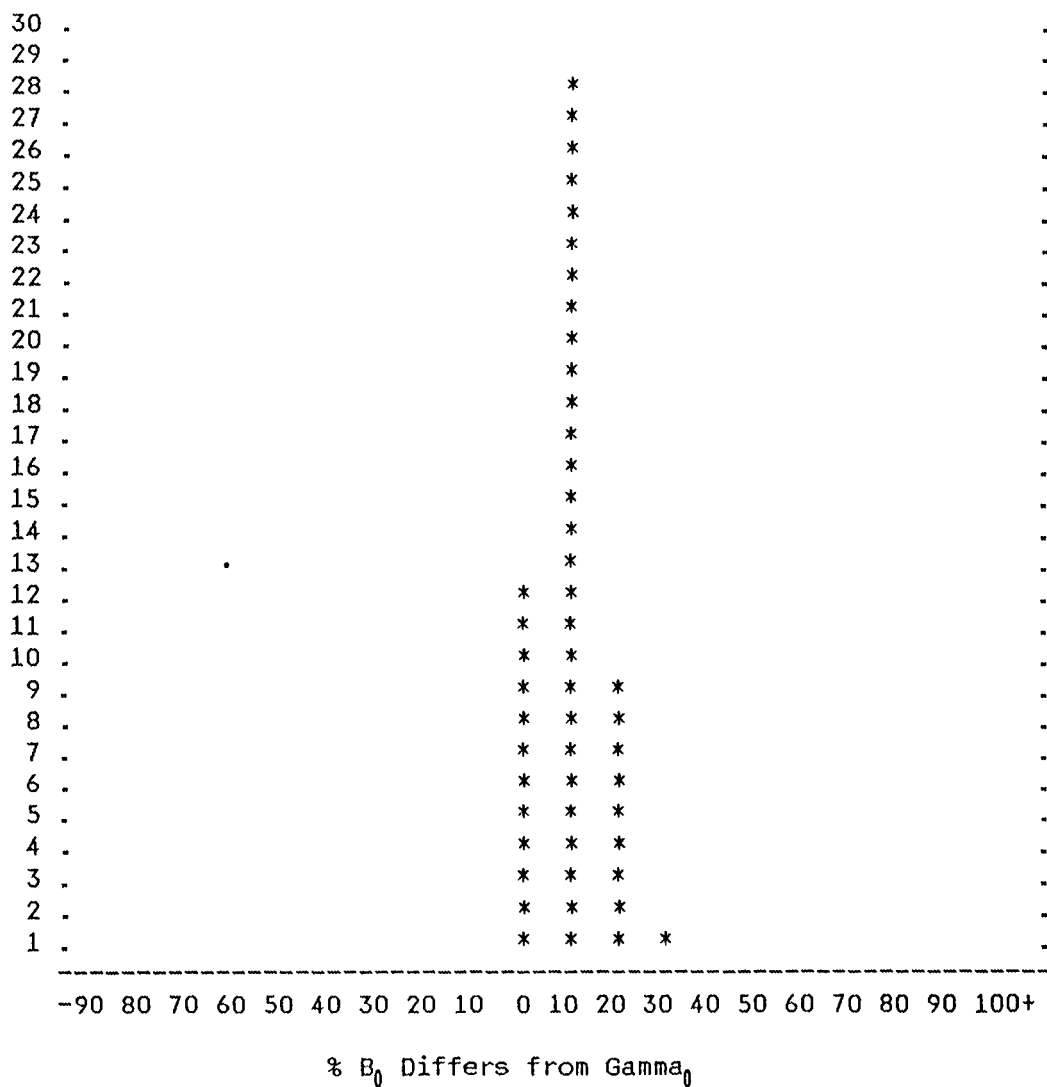


Chart of Estimates of B_0 with :

Gamma₀ = 50 Phi = 0.001
 n' = 45 r = 30

Source : 50out3
 Output File : 50out3.his

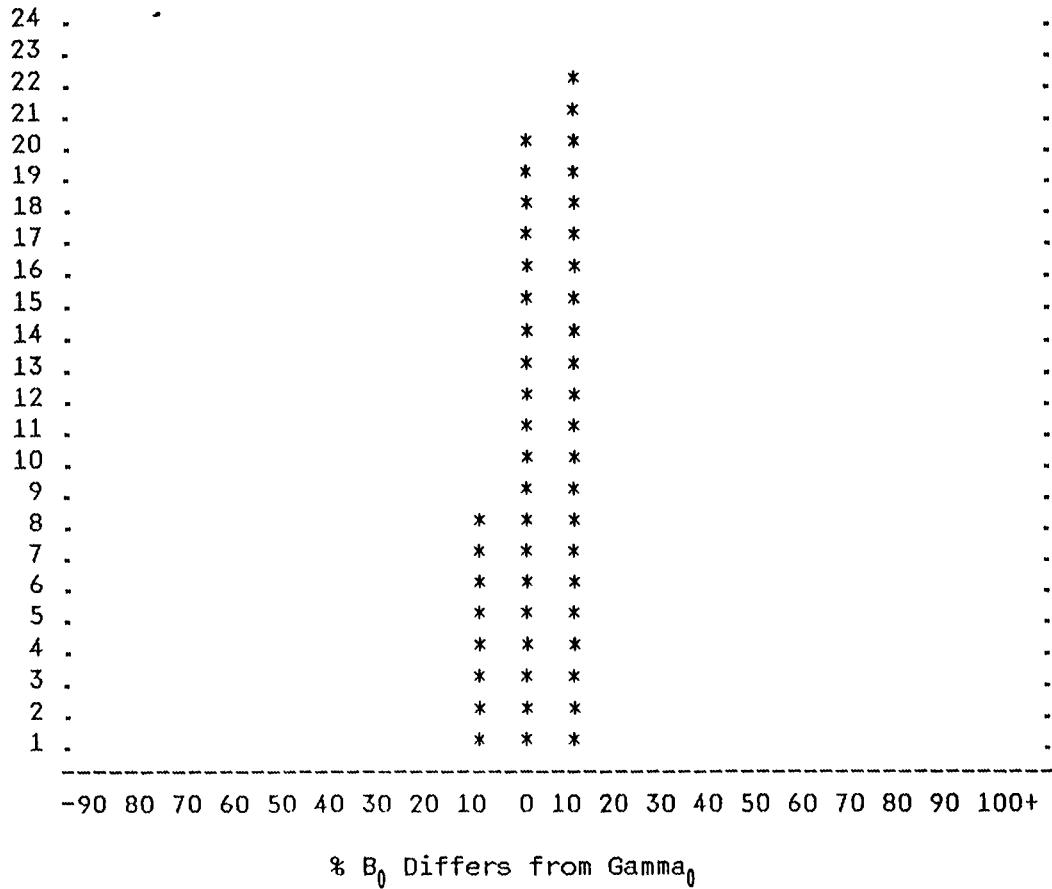
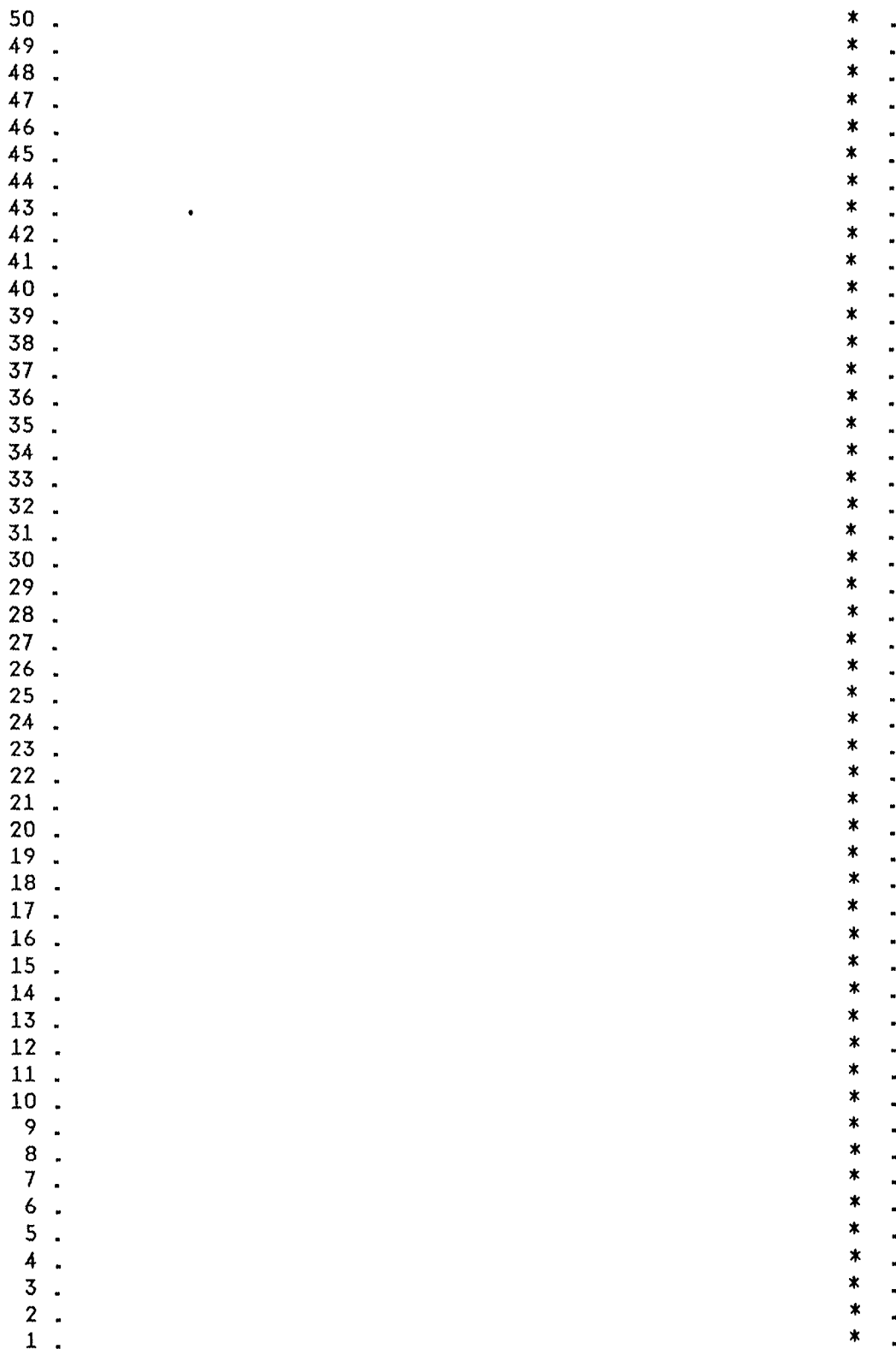


Chart of Estimates of B_0 with :

Gamma₀ = 100 Phi = 0.001
 n = 10 r = 30

Source : 100out30
 Output File : 100out30.his



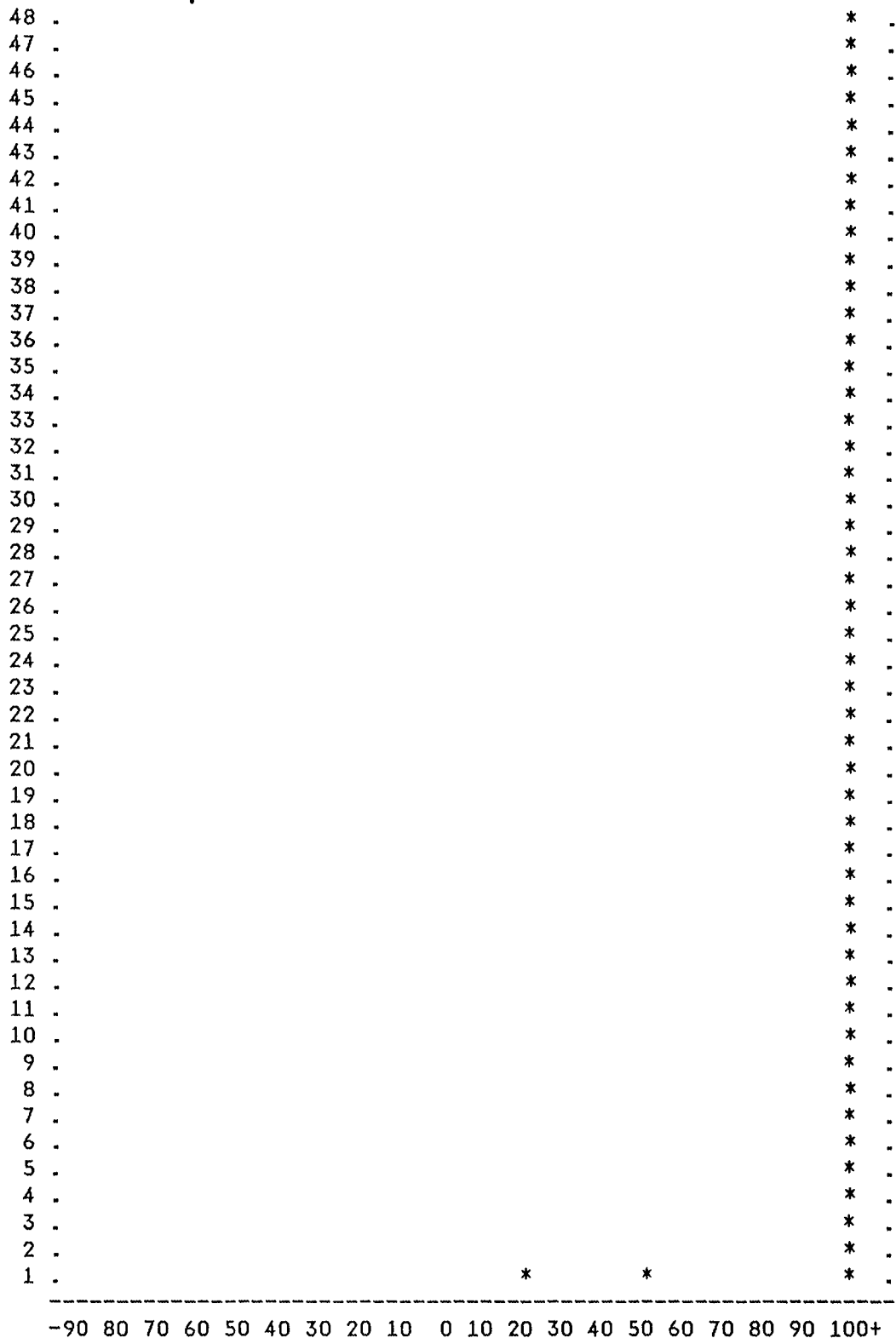
-90 80 70 60 50 40 30 20 10 0 10 20 30 40 50 60 70 80 90 100+

% B₀ Differs from Gamma₀

Chart of Estimates of B_0 with :

Gamma₀ = 100 Phi = 0.001
 n = 20 r = 30

Source : 100out30
 Output File : 100out30.his

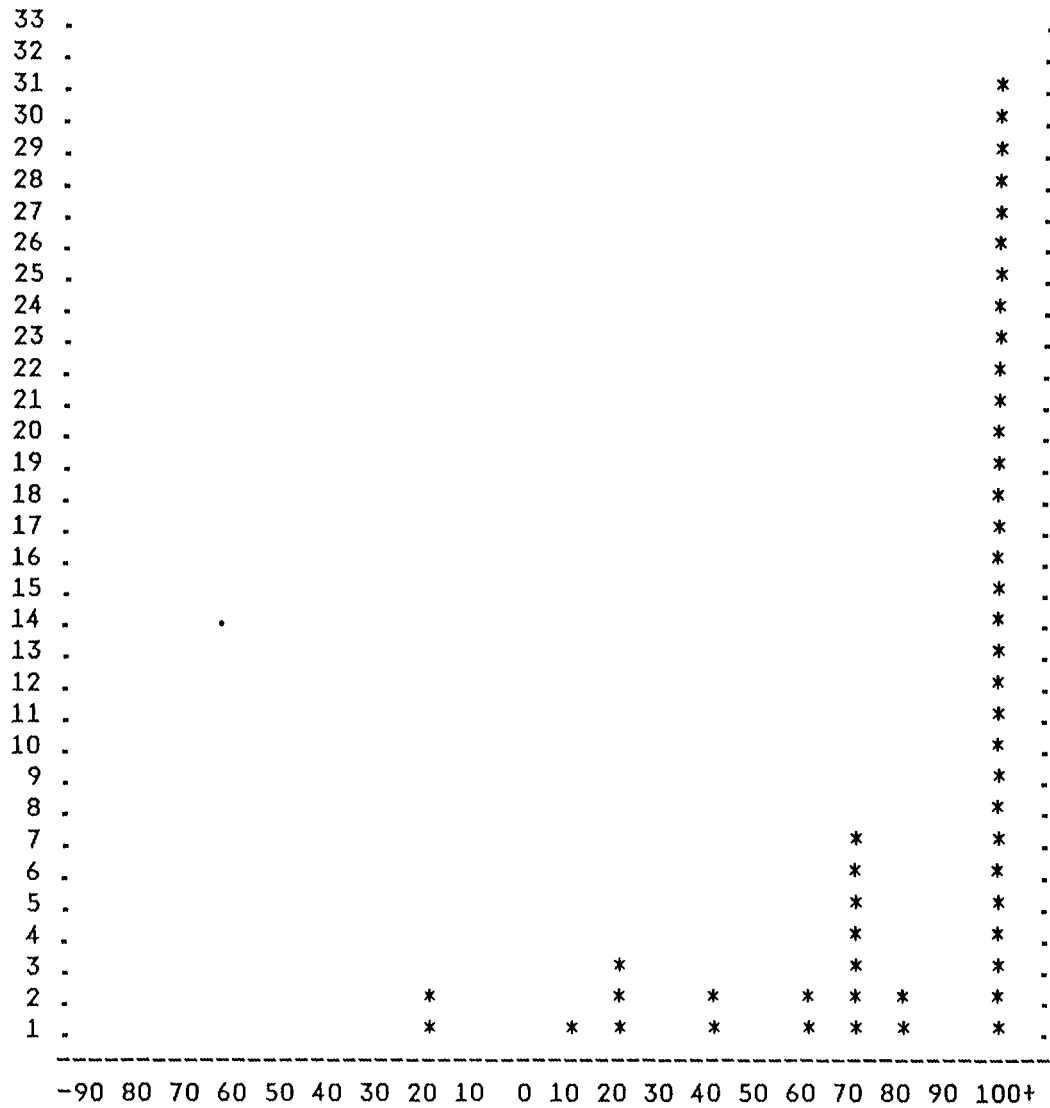


% B_0 Differs from Gamma₀

Chart of Estimates of B_0 with :

$\Gamma_0 = 100$ $\Phi = 0.001$
 $r = 30$ $r = 30$

Source : 100out30
 Output File : 100out30.his



% B_0 Differs from Γ_0

Chart of Estimates of B_0 with :

$\Gamma_0 = 100$ $\Phi = 0.001$
 $n = 40$ $r = 30$

Source : 100out30
 Output File : 100out30.his

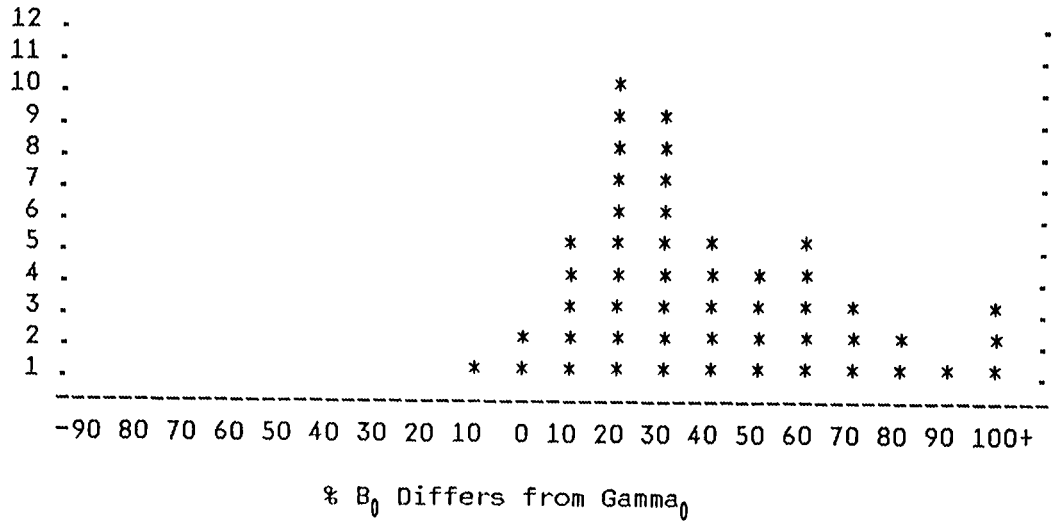


Chart of Estimates of B_0 with :

$\Gamma_0 = 100$ $\Phi = 0.001$
 $n = 50$ $r = 30$

Source : 100out30
 Output File : 100out30.his

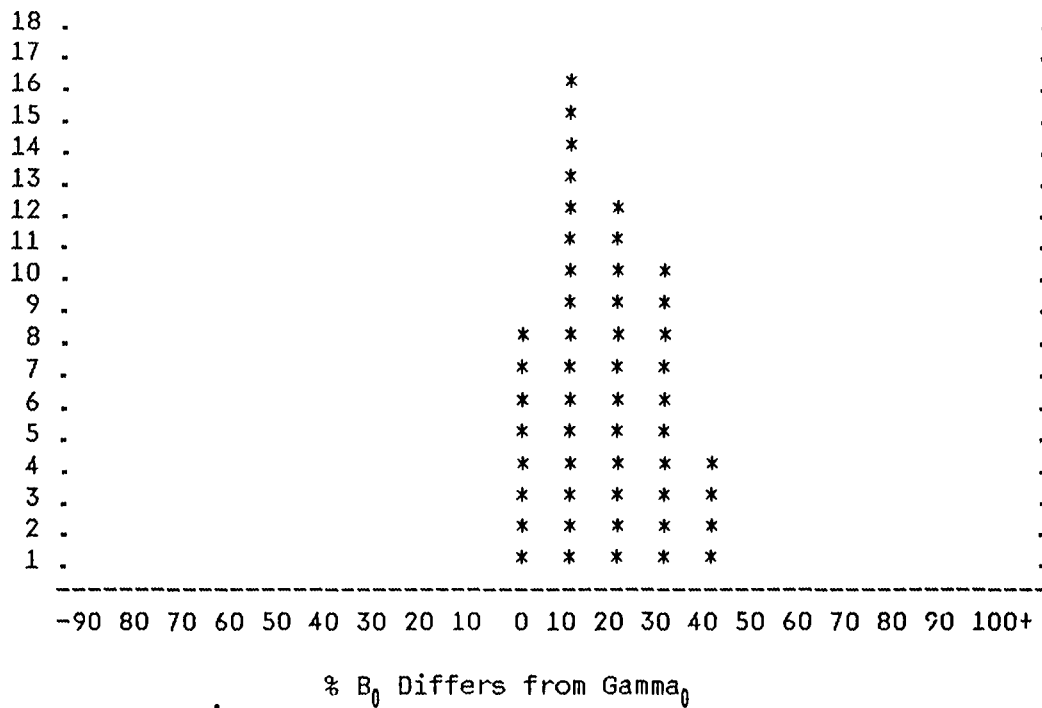
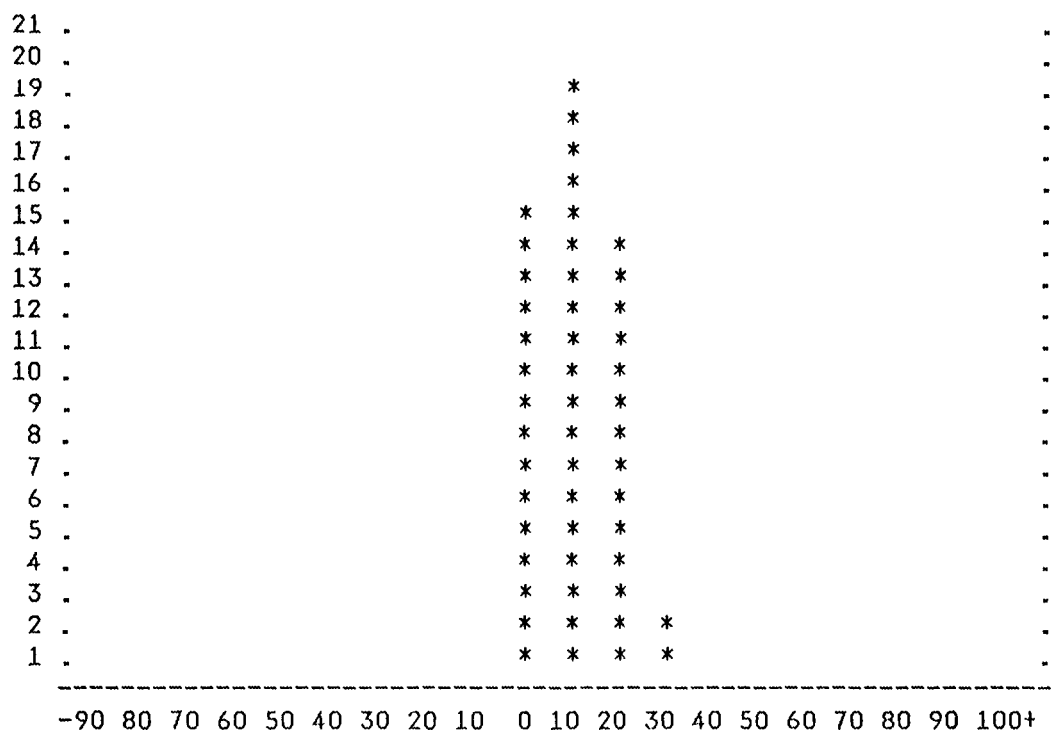


Chart of Estimates of B_0 with :

$\text{Gamma}_0 = 100$ $\text{Phi} = 0.001$
 $n = 60$ $r = 30$

Source : 100out30
 Output File : 100out30.his



. % B_0 Differs from Gamma_0

Chart of Estimates of B_0 with :

$\Gamma_0 = 100$ $\Phi = 0.001$
 $n = 70$ $r = 30$

Source : 100out30
 Output File : 100out30.his

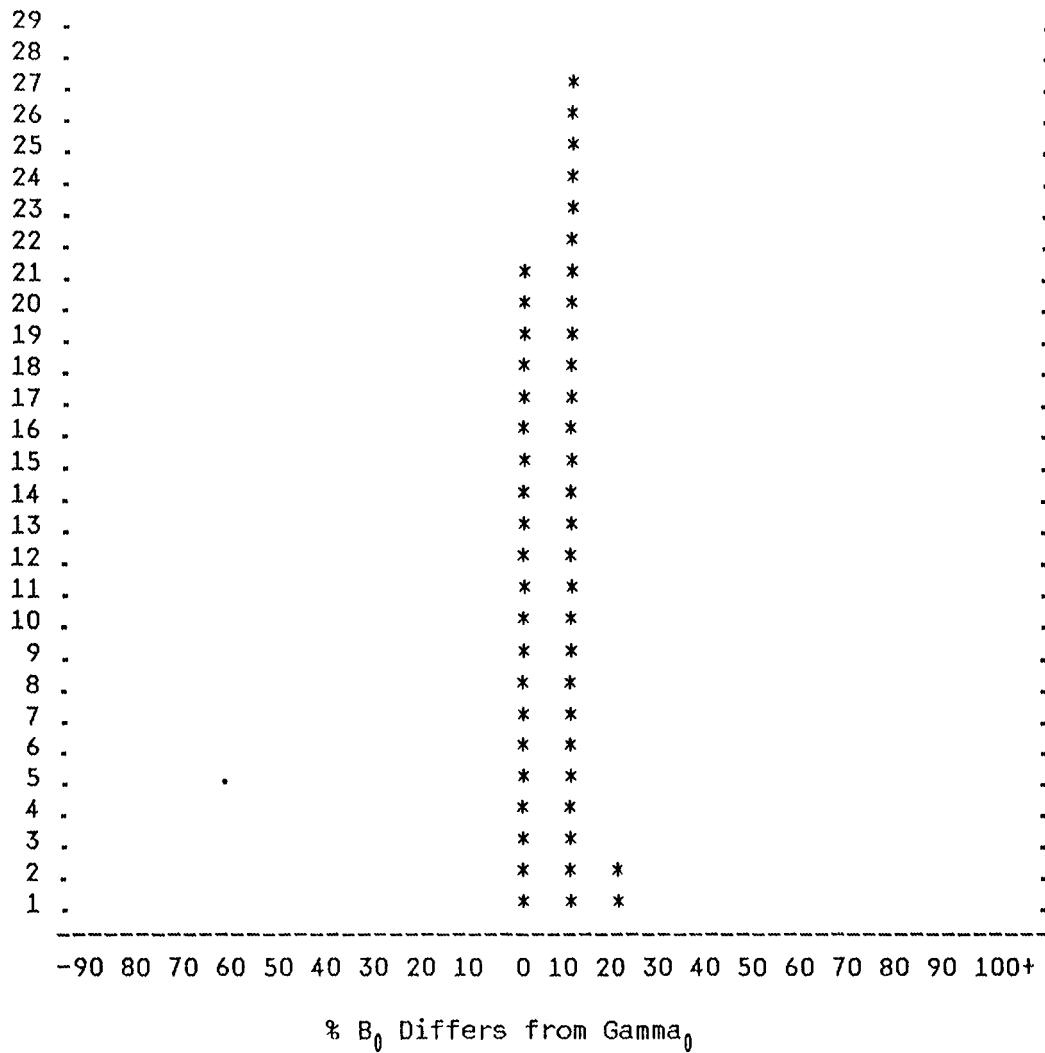
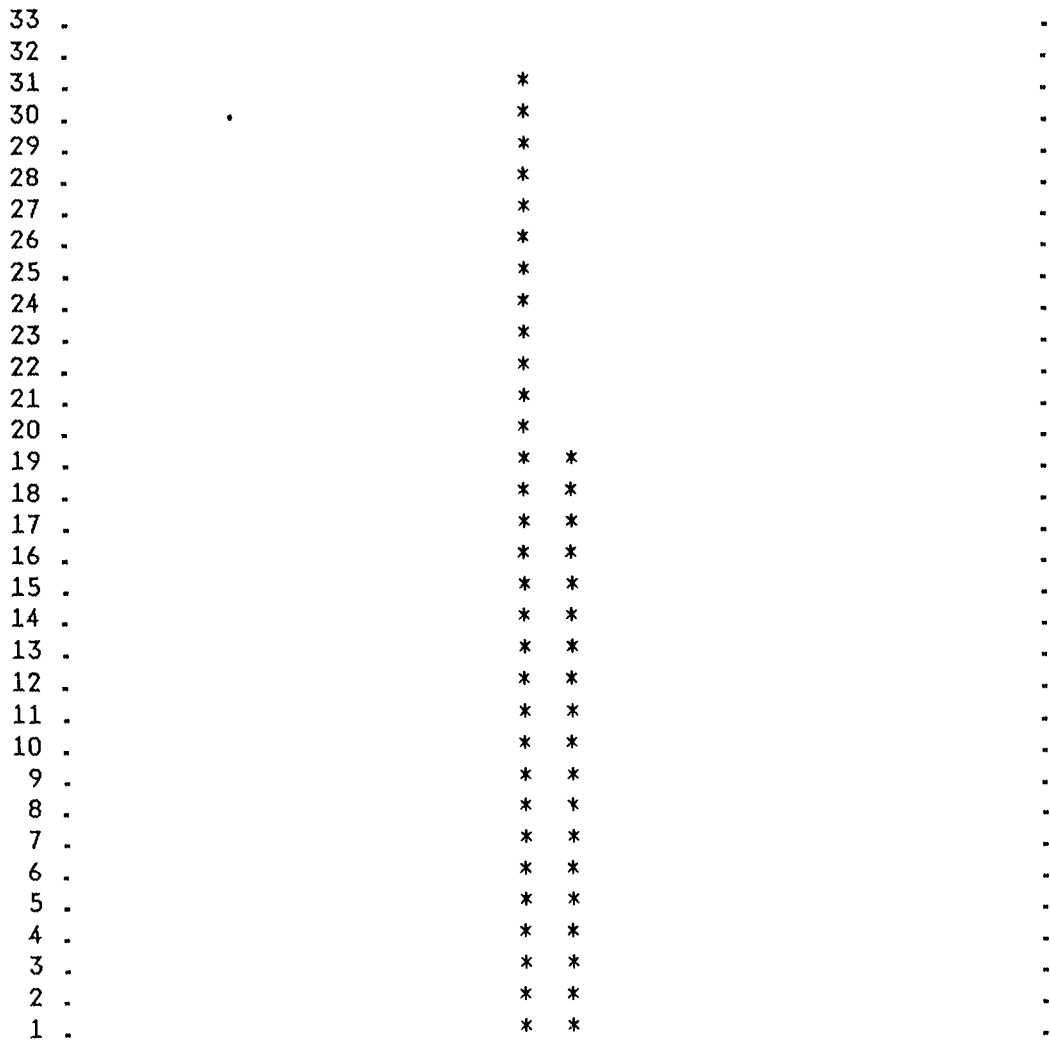


Chart of Estimates of B_0 with :

Gamma₀ = 100 Phi = 0.001
 n = 80 r = 30

Source : 100out30
 Output File : 100out30.his



-90 80 70 60 50 40 30 20 10 0 10 20 30 40 50 60 70 80 90 100+

% B_0 Differs from Gamma₀

Chart of Estimates of B_0 with :

$\Gamma_0 = 100$ $\Phi = 0.001$
 $n = 90$ $r = 30$

Source : 100out30
 Output File : 100out30.his

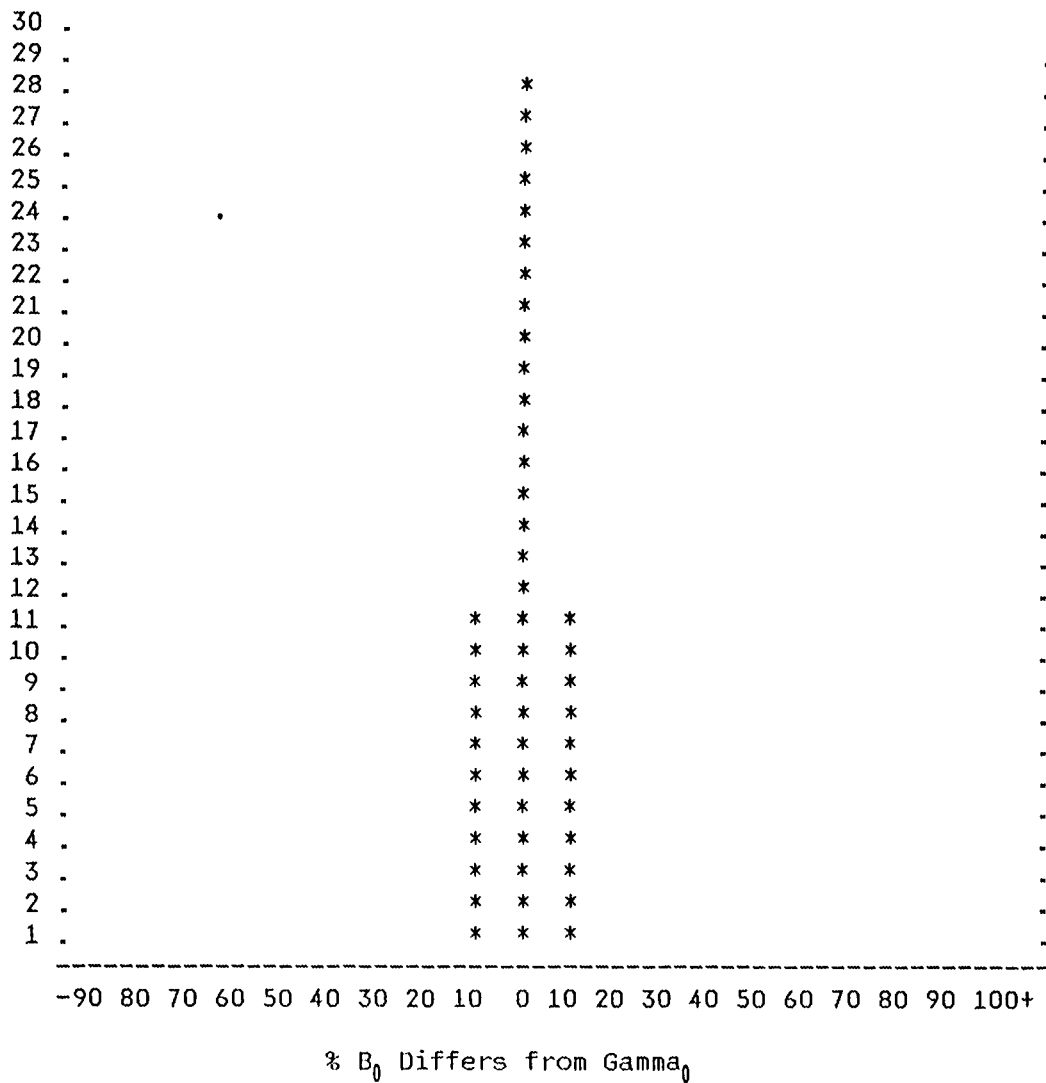
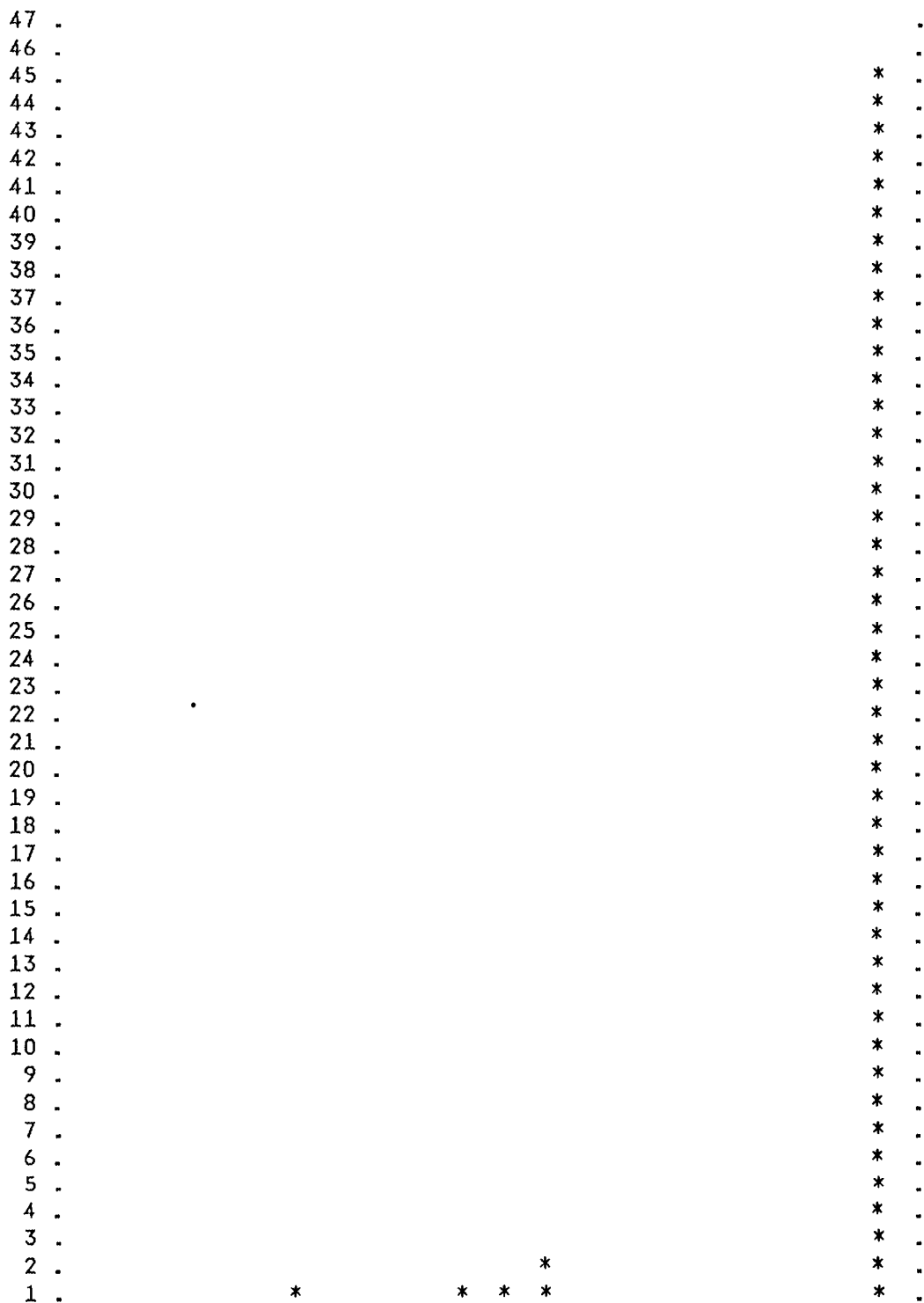


Chart of Estimates of B_0 with :

Gamma₀ = 200 Phi = 0.001
 n = 20 r = 30

Source : 200out30
 Output File : 200out30.his



-90 80 70 60 50 40 30 20 10 0 10 20 30 40 50 60 70 80 90 100+

% B_0 Differs from Gamma₀

Chart of Estimates of B_0 with :

Gamma₀ = 200 Phi = 0.001
 n = 40 r = 30

Source : 200out30
 Output File : 200out30.his

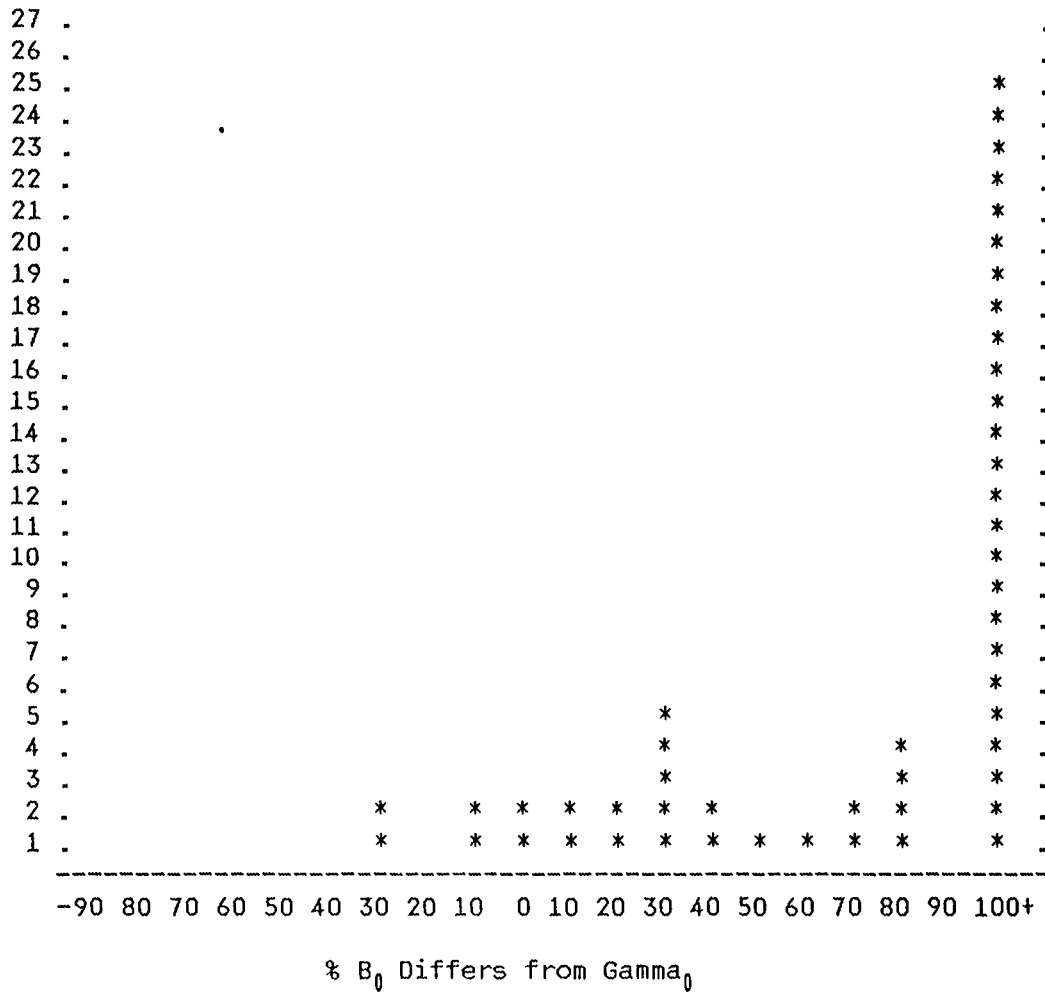


Chart of Estimates of B_0 with :

Gamma₀ = 200 Phi = 0.001
 n = 60 r = 30

Source : 200out30
 Output File : 200out30.his

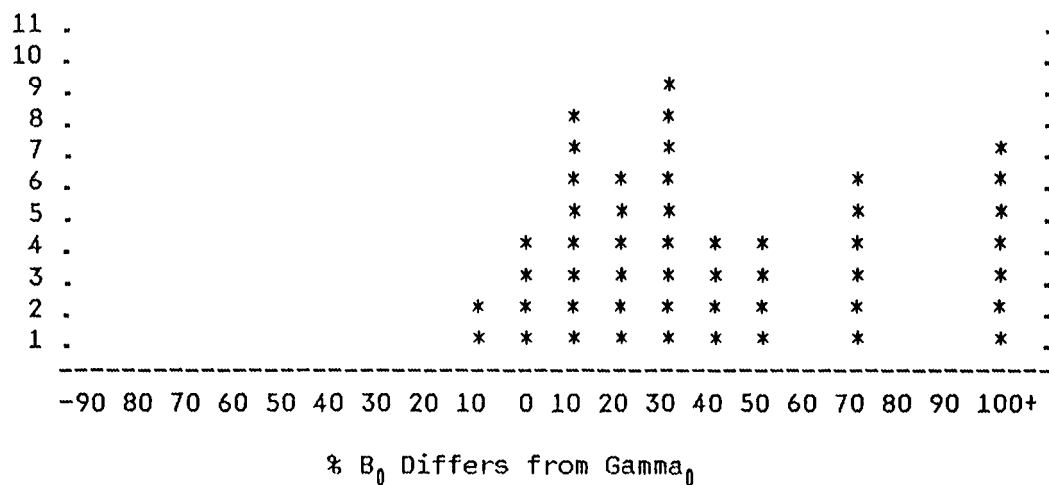


Chart of Estimates of B_0 with :

$\text{Gamma}_0 = 200$ $\text{Phi} = 0.001$
 $n = 80$ $r = 30$

Source : 200out30
 Output File : 200out30.his

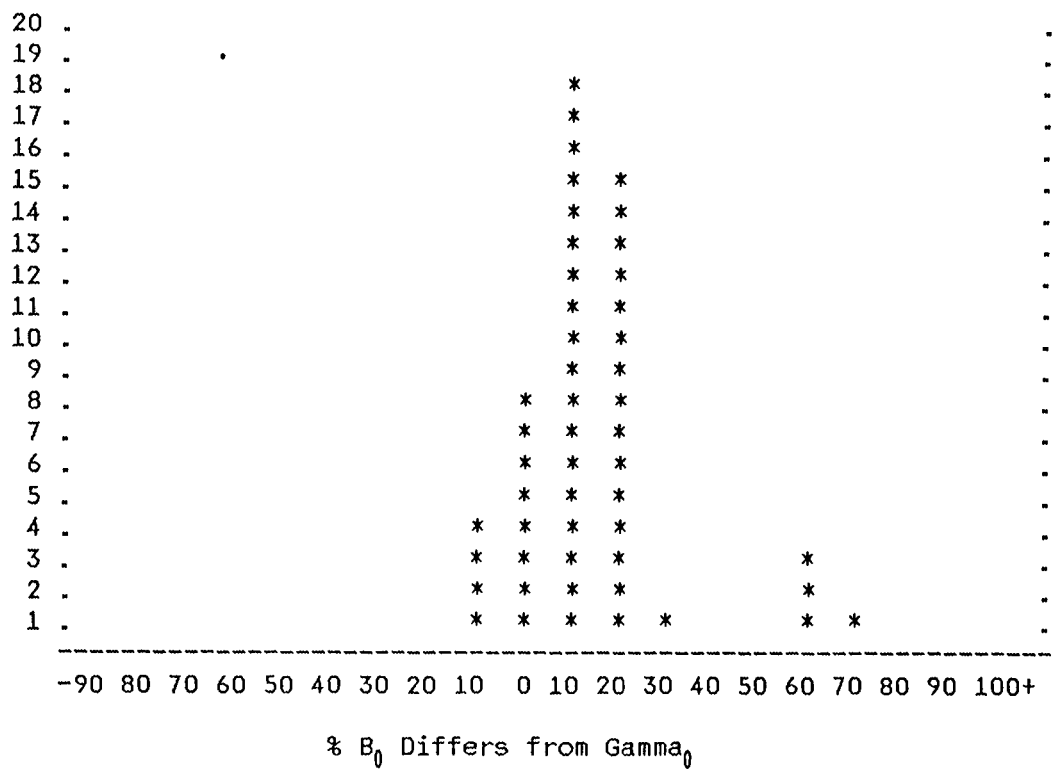


Chart of Estimates of B_0 with :

Gamma₀ = 200 Phi = 0.001
 n = 100 r = 30

Source : 200out30
 Output File : 200out30.his

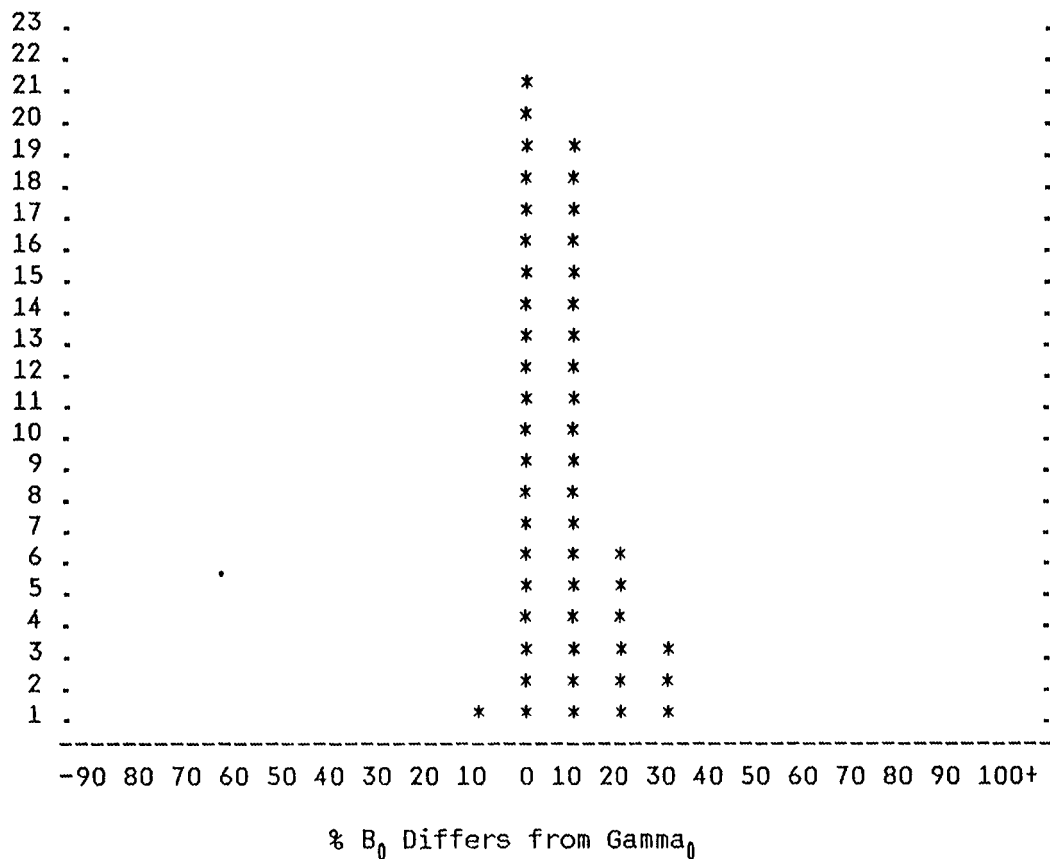


Chart of Estimates of B_0 with :

Gamma₀ = 200 Phi = 0.001
 n = 120 r = 30

Source : 200out30
 Output File : 200out30.his

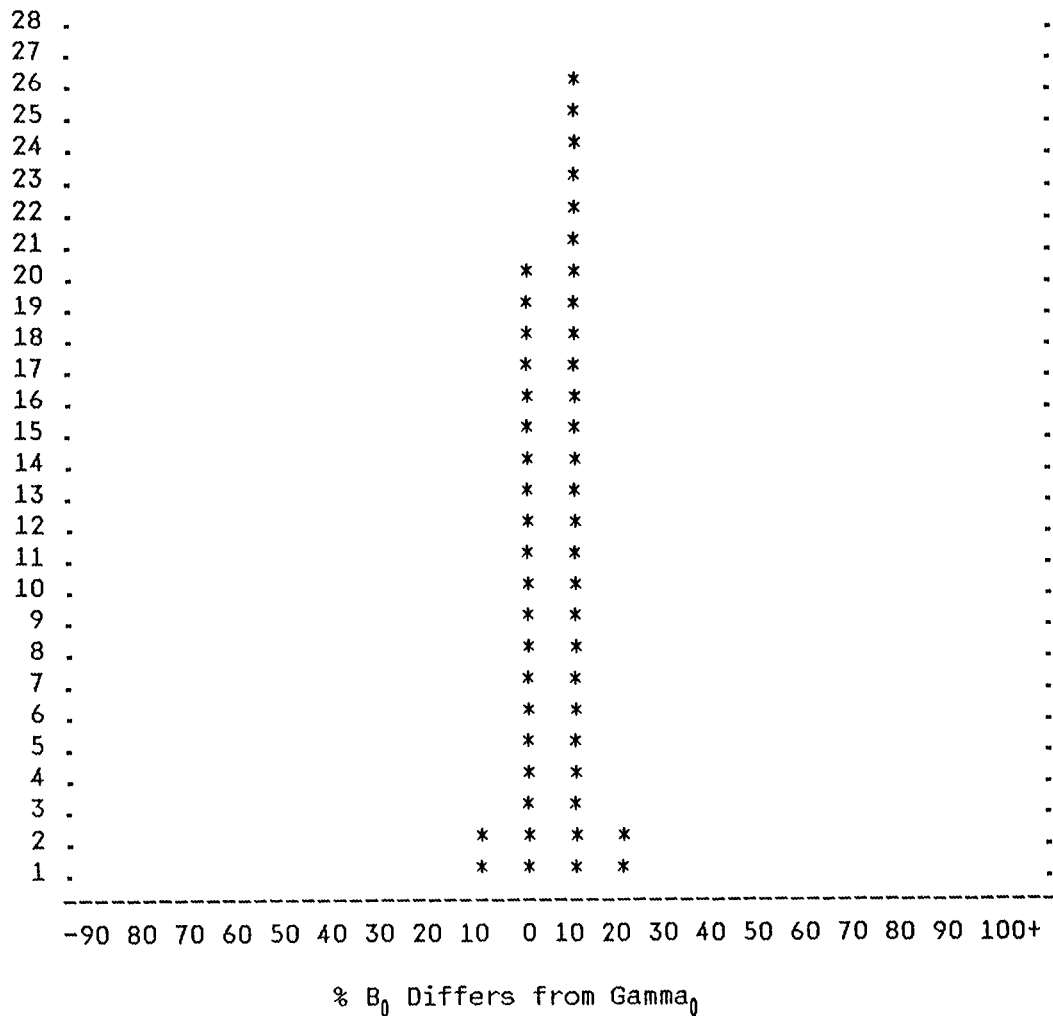
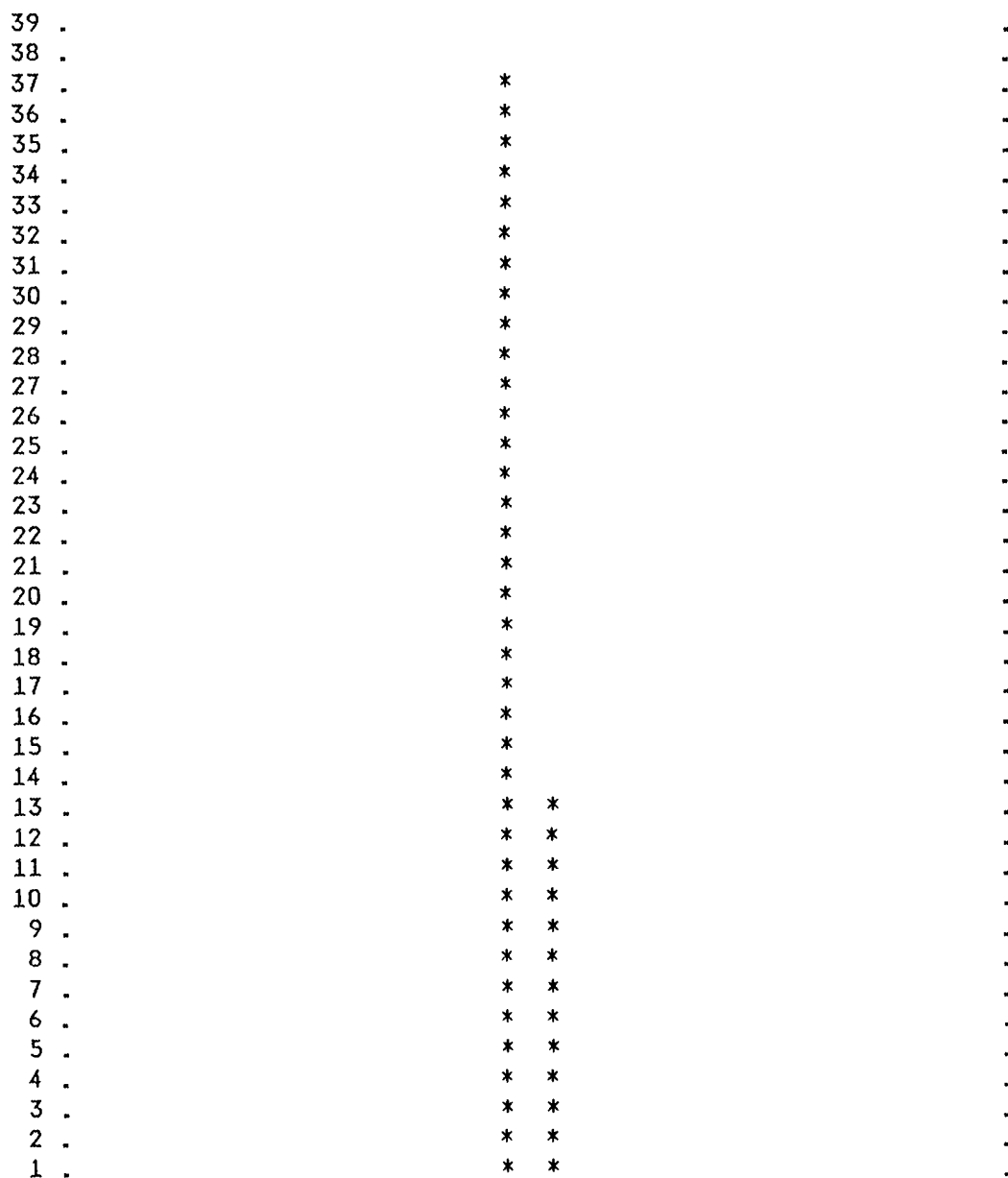


Chart of Estimates of B_0 with :

$\Gamma_0 = 200$ $\Phi = 0.001$
 $n = 140$ $r = 30$

Source : 200out30
 Output File : 200out30.his



-90 80 70 60 50 40 30 20 10 0 10 20 30 40 50 60 70 80 90 100+

% B_0 Differs from Γ_0

Chart of Estimates of B_0 with :

Gamma₀ = 200 Phi = 0.001
 n = 160 r = 30

Source : 200out30
 Output File : 200out30.his

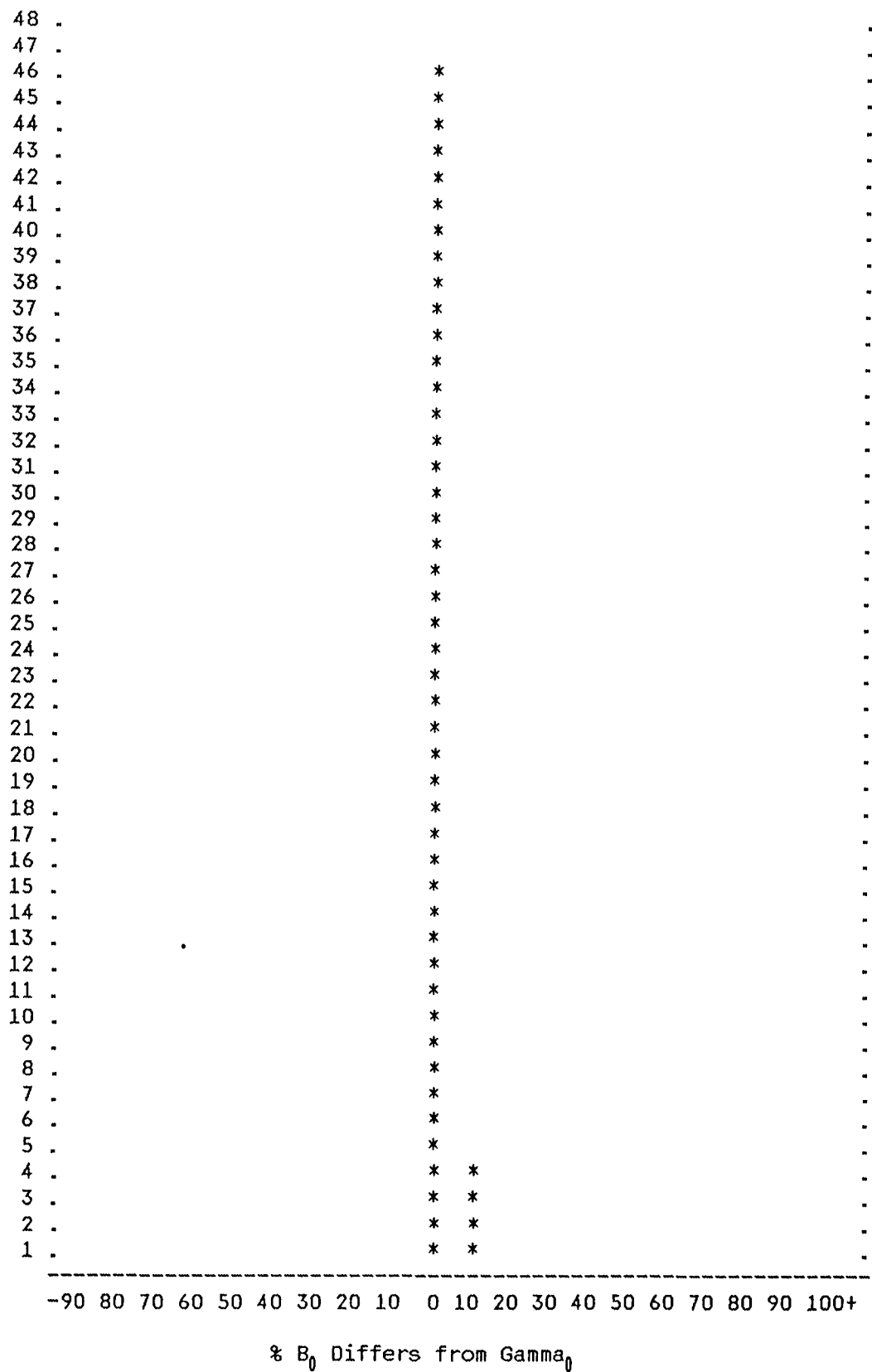
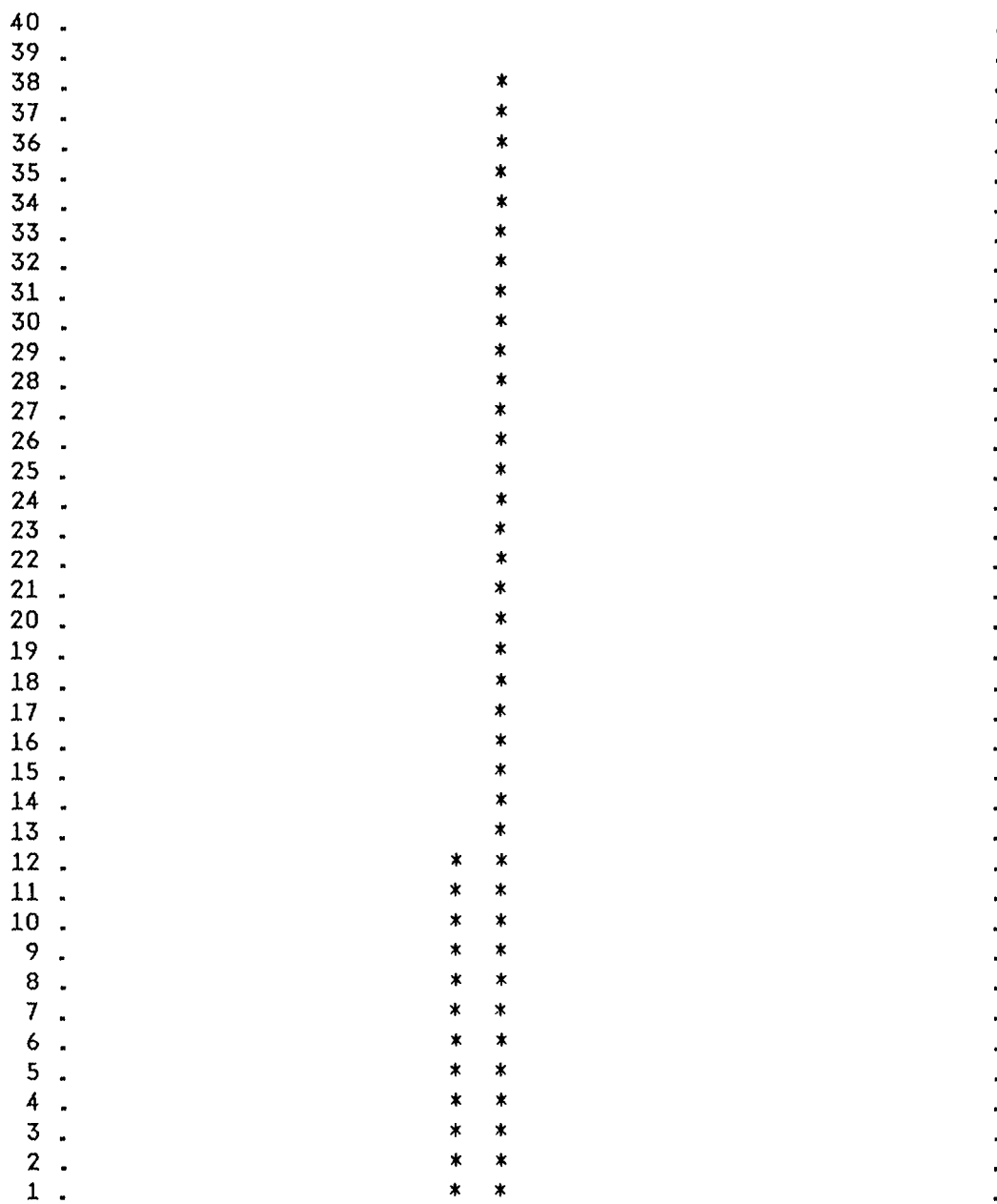


Chart of Estimates of B_0 with :

$\Gamma_0 = 200$ $\Phi = 0.001$
 $n = 180$ $r = 30$

Source : 200out30
Output File : 200out30.his



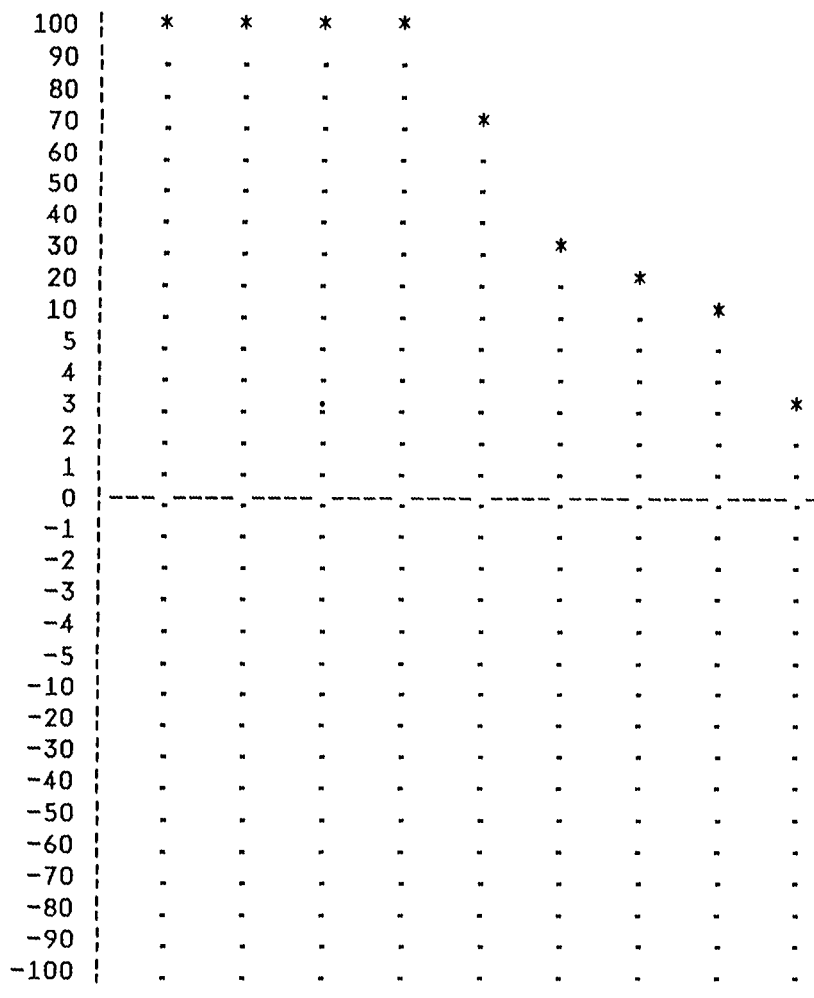
% B_0 Differs from Γ_0

Summary Graph of % Mean B_0 Estimate differs from Γ_{B_0}

$\Gamma_{B_0} = 50$, $\Phi = 0.001$
 $r = 30$

Source : 50out3
 Output : 50out3.gph

%off



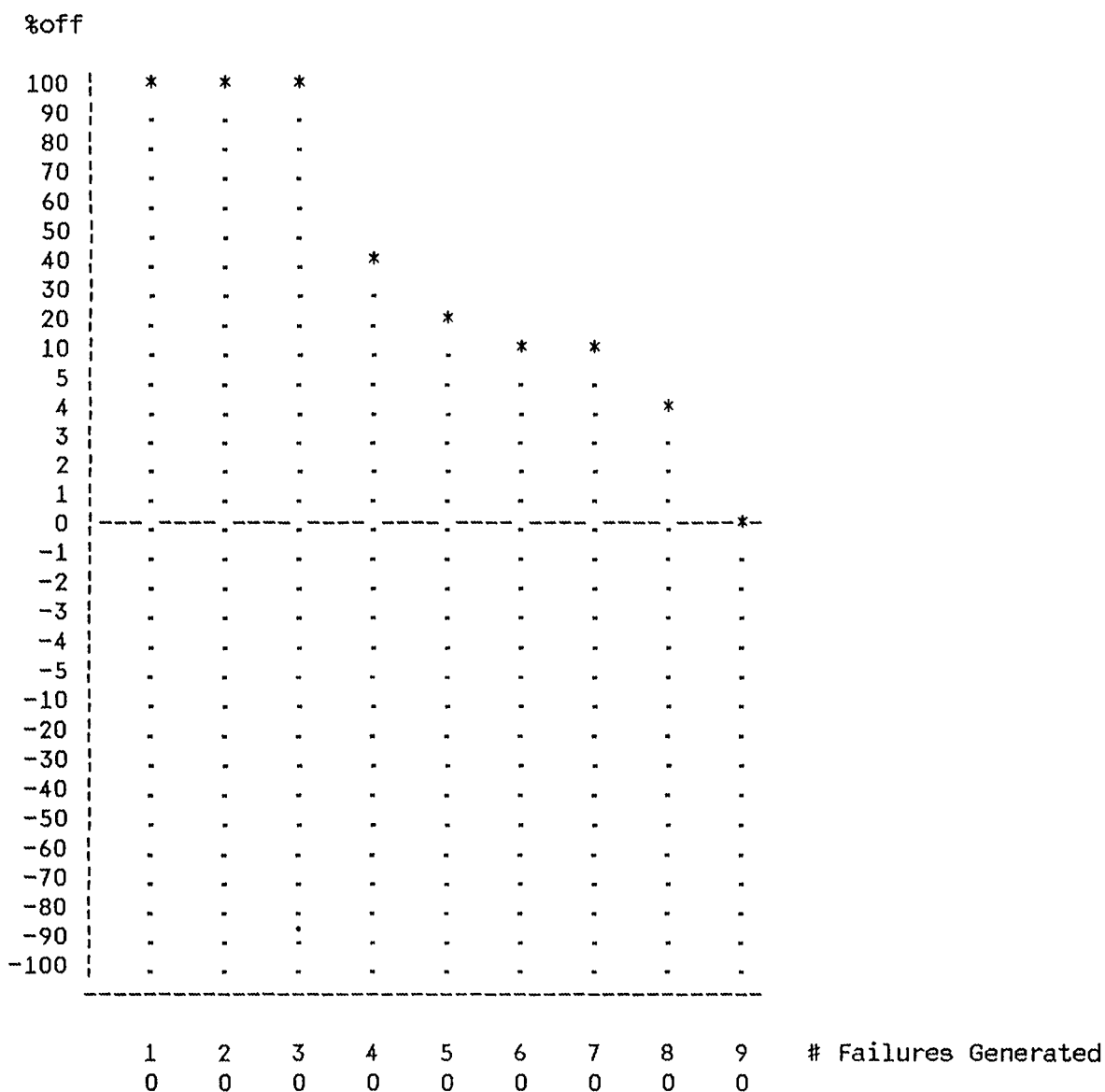
5 0 5 0 5 0 5 0 5 # Failures Generated

Me	b1		B_0		tsum	te
	Mean	Std	Mean	Std	Mean	Mean
5	0.00005000	0.00000000	970.76	68.93	308.0	103.8
10	0.00005000	0.00000000	935.34	57.75	1156.7	215.8
15	0.00010950	0.00010987	712.30	352.01	2611.2	343.0
20	0.00036750	0.00024379	292.73	316.88	4756.1	489.5
25	0.00060250	0.00019558	84.39	27.36	7703.6	660.1
30	0.00073825	0.00012294	65.02	8.55	11602.5	865.2
35	0.00082400	0.00009308	58.58	5.12	16680.4	1125.0
40	0.00089909	0.00007681	54.53	3.19	23350.8	1494.7
45	0.00097758	0.00008922	51.57	3.21	32898.0	2334.7

Summary Graph of % Mean B_0 Estimate differs from Γ_{B_0}

$\Gamma_{B_0} = 100$, $\Phi = 0.001$
 $r = 30$

Source : 100out30
 Output : 100out30.gph



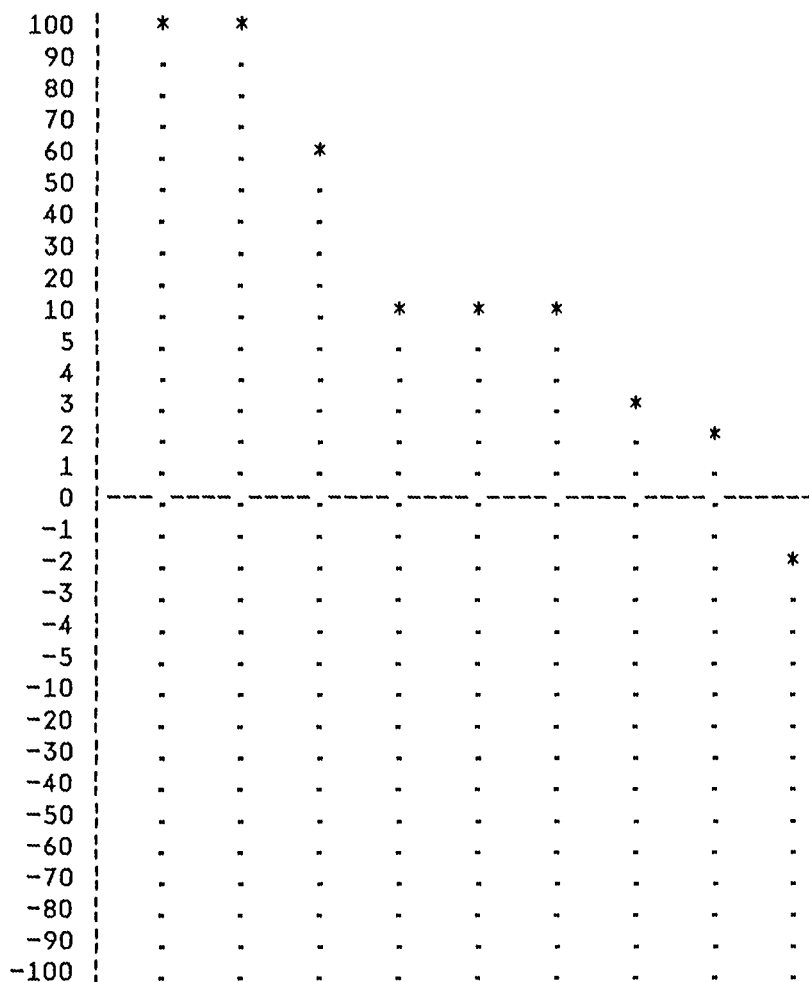
Me	b1		B_0		tsum	te
	Mean	Std	Mean	Std	Mean	Mean
10	0.00005000	0.00000000	1951.87	115.25	556.0	103.1
20	0.00010800	0.00016624	1616.19	579.77	2202.1	216.7
30	0.00045300	0.00030223	454.98	645.05	5072.1	346.8
40	0.00071300	0.00016591	141.31	32.89	9353.7	497.0
50	0.00083275	0.00010148	117.93	11.56	15274.8	673.8
60	0.00089131	0.00007702	110.48	7.36	23148.6	887.7
70	0.00092544	0.00005295	106.71	4.87	33470.5	1160.7
80	0.00095044	0.00004071	104.41	4.02	47042.0	1541.2
90	0.00101808	0.00010291	99.78	5.79	68463.0	3621.2

Summary Graph of % Mean B_0 Estimate differs from Γ_{Γ_0}

$\Gamma_{\Gamma_0} = 200$, $\Phi = 0.001$
 $r = 30$

Source : 200out30
 Output : 200out30.gph

%off



2 4 6 8 1 1 1 1 1
 0 0 0 0 0 0 0 0 0 # Failures Generated

Me	b1		B_0		tsum	te
	Mean	Std	Mean	Std	Mean	Mean
20	0.00016200	0.00033205	3429.13	1189.48	1068.5	104.0
40	0.00048050	0.00040888	1431.32	1536.65	4350.3	220.5
60	0.00072600	0.00021572	310.90	185.42	10113.2	352.1
80	0.00087312	0.00014022	229.90	34.92	18719.8	504.9
100	0.00091719	0.00008909	216.31	16.76	30642.0	683.9
120	0.00093897	0.00006673	211.06	10.22	46510.1	900.8
140	0.00096191	0.00004103	206.53	5.69	67333.1	1180.9
160	0.00097996	0.00003572	203.67	4.69	94830.9	1576.5
180	0.00104322	0.00008510	196.57	8.29	135774.6	3681.0

Appendix C

Appendix C contains summary graphs which show the affect of increasing the number of replications for a given number of failure times with each total fault count used. An Appendix C summary graph shows the change in the mean of 50 B_0 parameter estimations for one trial size with the different number of data replications used.

APPENDIX C INDEX

C1-1	Summary Graphs for 50 total faults
C2-1	Summary Graphs for 100 total faults
C3-1	Summary Graphs for 200 total faults

Summary Graph of % Mean B_0 Estimate differs from Γ_0

$\Gamma_0 = 50$ $\Phi_1 = 0.001$
 $n = 5$

Source : 50grp5
 Outout : 50grp5.gph

%off

100	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*
90
80
70
60
50
40
30
20
10
5
4
3
2
1
0
-1
-2
-3
-4
-5
-10
-20
-30
-40
-50
-60
-70
-80
-90
-100

						1	2	3	4	5	6	7	8	9	1	
						0	0	0	0	0	0	0	0	0	0	# replicates
1	2	3	4	5	0	0	0	0	0	0	0	0	0	0	0	

rep	b1		b0		tsum	te
	Mean	Std	Mean	Std	Mean	Mean
1	0.00136750	0.00367522	1248.42	1488.88	304.5	105.9
2	0.00113000	0.00263317	905.71	606.50	312.7	104.7
3	0.00025800	0.00060294	930.46	448.45	304.1	104.4
4	0.00023000	0.00078537	1003.19	334.68	292.7	98.8
5	0.00027400	0.00122304	1009.53	290.59	292.2	98.9
10	0.00005000	0.00000000	1007.09	128.00	298.1	101.3
20	0.00005000	0.00000000	999.29	94.21	302.3	101.3
30	0.00005000	0.00000000	970.76	68.93	308.0	103.8
40	0.00005000	0.00000000	995.57	58.62	296.0	101.1
50	0.00005000	0.00000000	1004.18	68.87	297.5	100.3
60	0.00005000	0.00000000	989.67	64.39	301.4	101.7
70	0.00005000	0.00000000	975.51	46.92	305.7	103.0
80	0.00005000	0.00000000	992.25	56.02	298.8	101.4
90	0.00005000	0.00000000	994.91	48.89	299.3	101.0
100	0.00005000	0.00000000	971.85	46.42	306.7	103.4

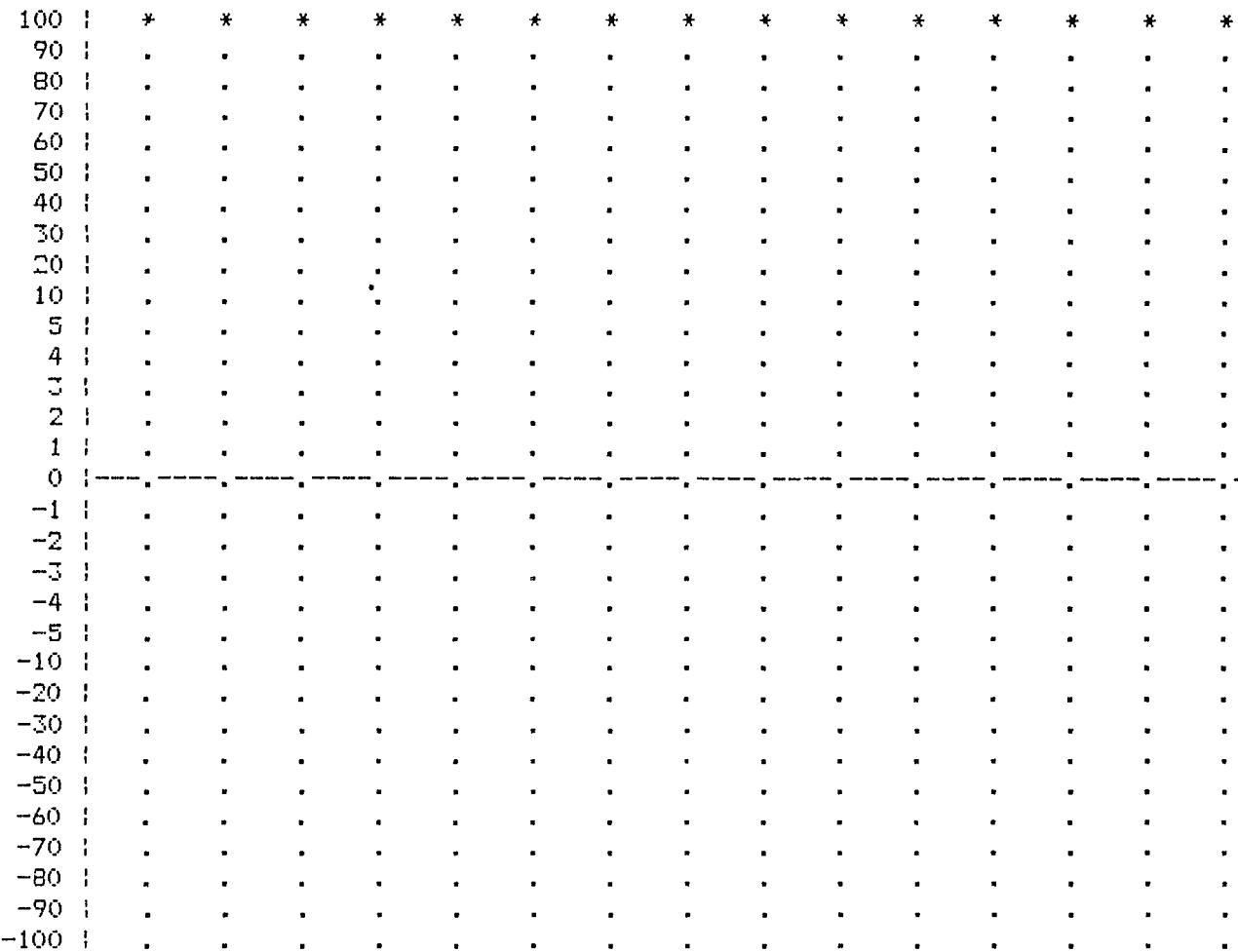
Summary Graph of % Mean B_0 Estimate differs from Γ_0

(

$\Gamma_0 = 50$ $Ph_1 = 0.001$
 $n = 10$

Source : 50grp10
 Output : 50grp10.gph

%off



replicates

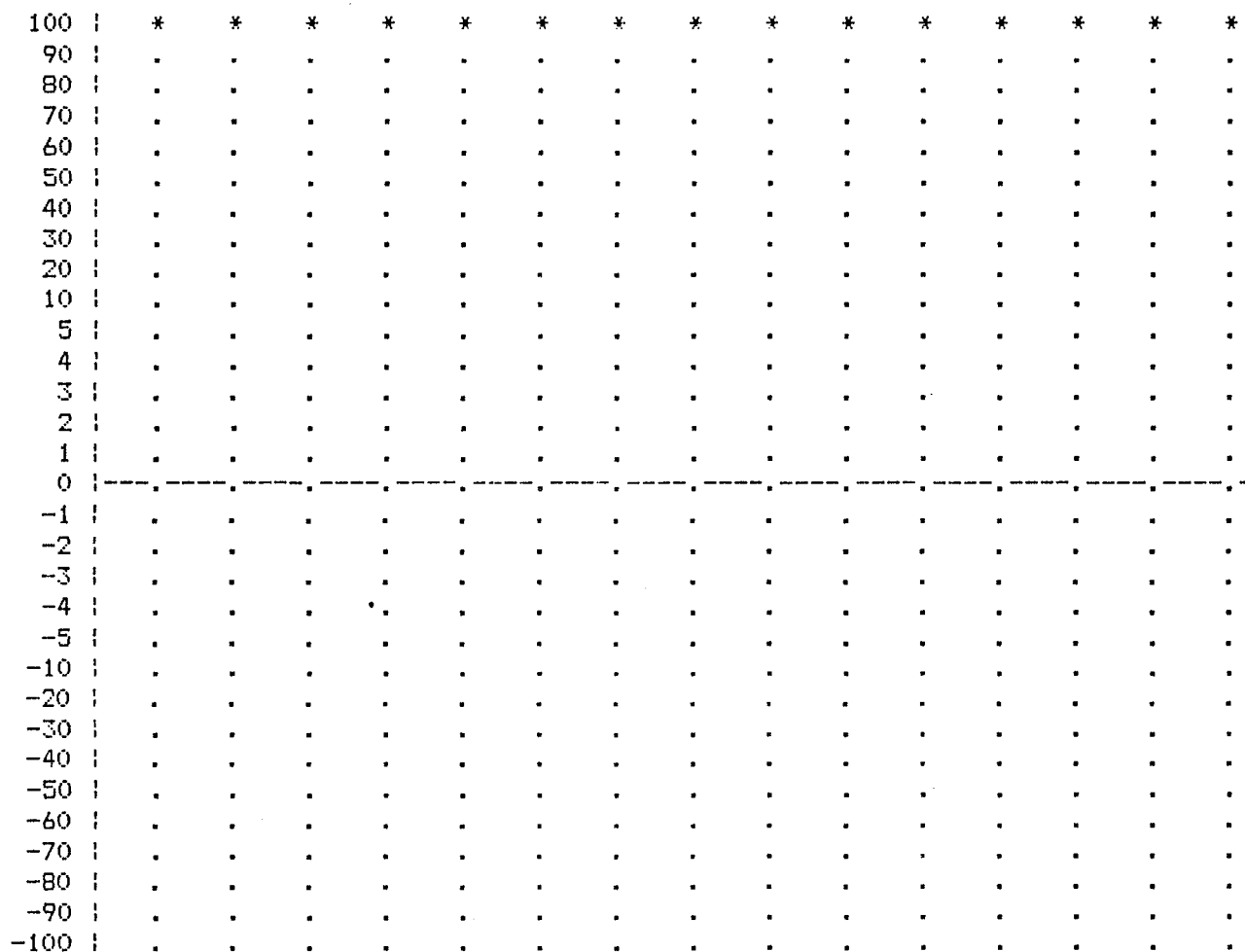
rep	b1		b0		tsum	te
	Mean	Std	Mean	Std	Mean	Mean
1	0.00147675	0.00279149	884.04	829.23	1207.7	227.1
2	0.00080875	0.00228739	1013.25	693.15	1133.8	208.5
3	0.00046600	0.00086994	761.53	493.28	1190.4	226.0
4	0.00031800	0.00080624	903.14	432.19	1122.3	211.4
5	0.00060300	0.00099412	797.03	589.05	1132.1	216.8
10	0.00013900	0.00032604	898.12	335.02	1139.0	214.3
20	0.00008600	0.00014664	884.31	213.52	1149.4	216.7
30	0.00005000	0.00000000	935.34	57.75	1156.7	215.8
40	0.00005000	0.00000000	940.59	43.28	1138.7	214.2
50	0.00005400	0.00002800	928.86	118.95	1133.7	214.0
60	0.00005000	0.00000000	936.21	47.62	1146.1	215.3
70	0.00005000	0.00000000	935.93	32.94	1155.3	215.1
80	0.00005000	0.00000000	935.31	35.71	1143.7	215.3
90	0.00005000	0.00000000	938.68	35.42	1139.6	214.5
100	0.00005000	0.00000000	931.10	36.29	1157.4	216.3

Summary Graph of % Mean B_0 Estimate differs from Γ_0

$\Gamma_0 = 50$ $\Phi = 0.001$
 $n = 15$

Source : 50grp15
 Output : 50grp15.gph

%off



						1	2	3	4	5	6	7	8	9	10	# replicates
1	2	3	4	5	0	0	0	0	0	0	0	0	0	0	0	

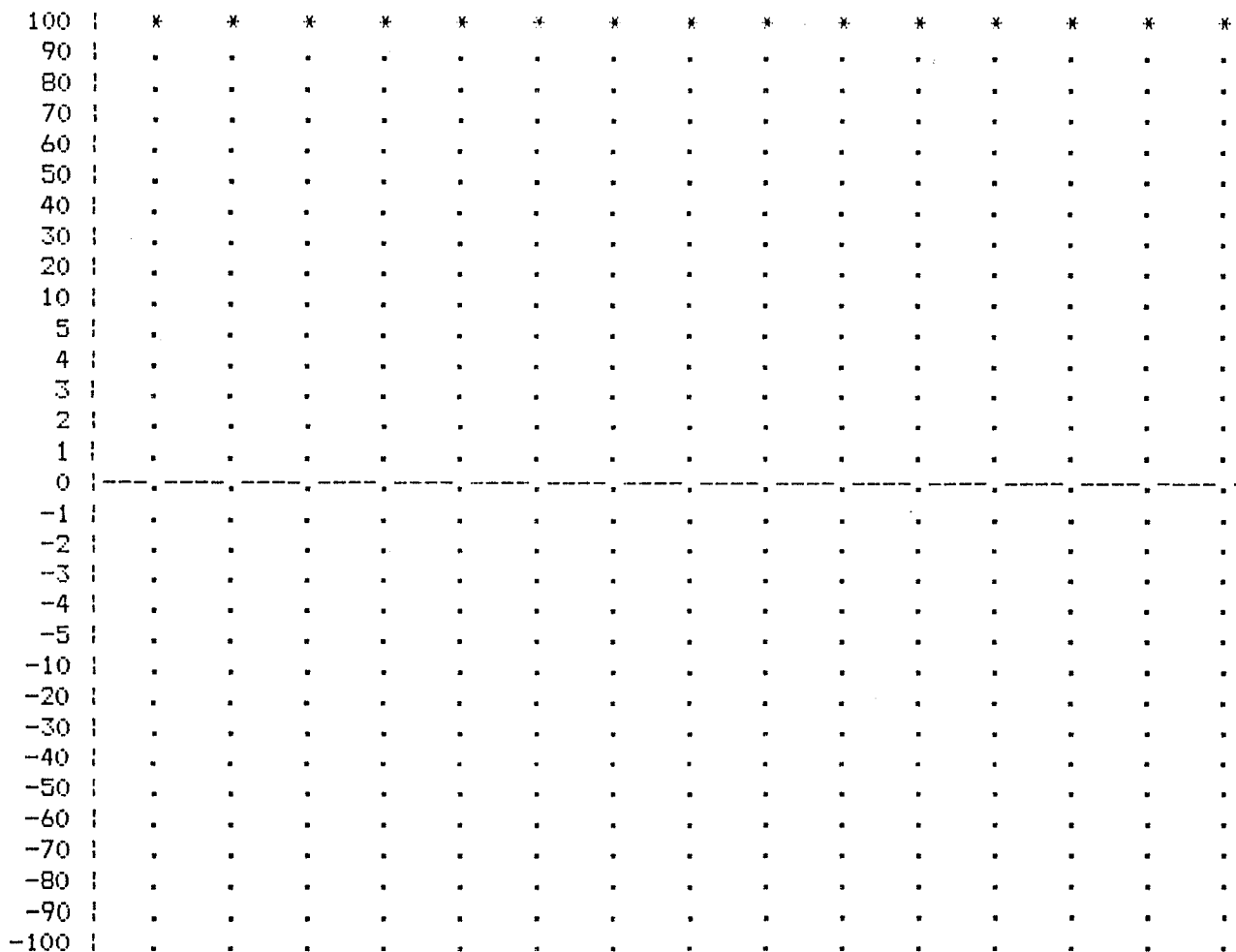
rep	b1		b0		tsum	te
	Mean	Std	Mean	Std	Mean	Mean
1	0.00100312	0.00165105	1453.48	1894.83	2763.1	359.8
2	0.00081550	0.00122681	897.07	952.06	2552.4	337.9
3	0.00032825	0.00062636	1020.48	945.15	2694.0	349.7
4	0.00031550	0.00050965	866.24	737.50	2534.5	331.9
5	0.00054850	0.00065419	616.44	726.86	2602.0	348.2
10	0.00021950	0.00031262	728.56	518.79	2583.7	340.1
20	0.00019550	0.00023631	613.09	382.03	2613.6	346.4
30	0.00010950	0.00010987	712.30	352.01	2611.2	343.0
40	0.00014600	0.00017772	671.10	341.21	2592.6	344.1
50	0.00010000	0.00012042	751.41	285.39	2577.8	340.4
60	0.00010800	0.00012975	722.39	290.29	2606.0	344.9
70	0.00006600	0.00004176	798.30	210.68	2611.1	343.4
80	0.00008600	0.00008663	745.93	269.21	2594.8	342.8
90	0.00007400	0.00008139	806.93	217.51	2585.5	341.6
100	0.00006000	0.00004583	842.95	153.96	2613.6	343.5

Summary Graph of % Mean B_0 Estimate differs from Γ_0

$\Gamma_0 = 50$ $\Phi = 0.001$
 $n = 20$

Source : 50grp20
 Outout : 50grp20.gph

%off



					1											1								
					0		2		3		4		5		6		7		8		9		0	
1	2	3	4	5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

replicates

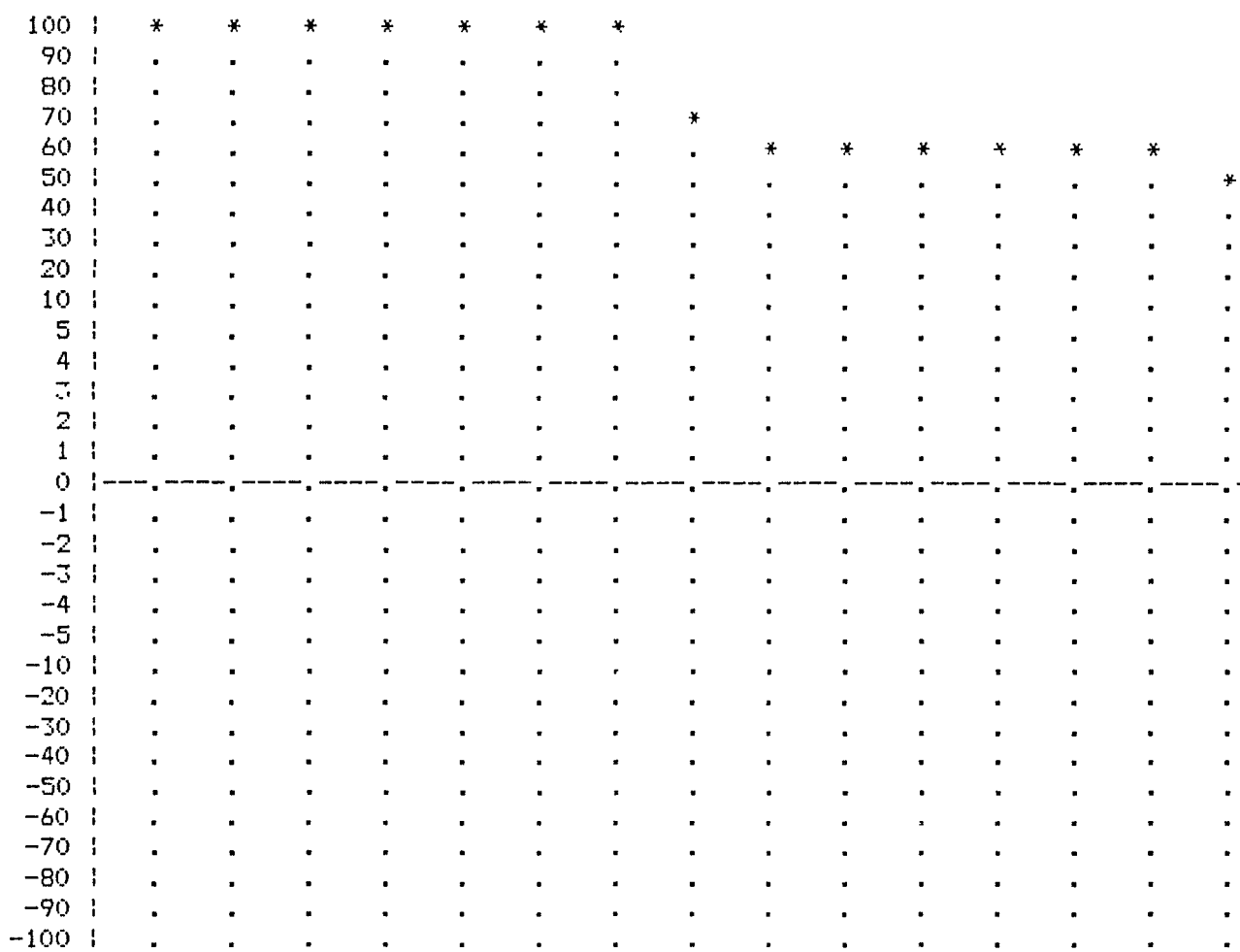
rep	b1		b0		tsum	te
	Mean	Std	Mean	Std	Mean	Mean
1	0.00087137	0.00109232	1168.41	1885.41	4986.2	514.7
2	0.00070950	0.00092951	825.97	1092.19	4650.6	479.2
3	0.00033900	0.00044347	700.53	755.55	4861.8	492.3
4	0.00054675	0.00057352	483.90	614.99	4640.9	481.8
5	0.00065750	0.00057373	394.12	580.91	4794.4	503.1
10	0.00045050	0.00039365	378.84	410.99	4716.9	487.1
20	0.00042750	0.00027264	245.49	307.16	4782.3	494.8
30	0.00036750	0.00024379	292.73	316.88	4756.1	489.5
40	0.00041650	0.00022724	194.30	222.72	4743.3	491.1
50	0.00038550	0.00023276	264.27	298.90	4706.8	485.8
60	0.00040400	0.00020008	181.56	206.63	4762.6	492.6
70	0.00034050	0.00017073	201.80	199.26	4758.2	489.5
80	0.00036900	0.00014548	163.66	145.35	4735.8	488.1
90	0.00039600	0.00016472	157.43	148.34	4724.3	488.3
100	0.00035600	0.00015553	180.53	170.46	4765.5	490.8

Summary Graph of % Mean B_0 Estimate differs from Γ_0

$\Gamma_0 = 50$ $\Phi_1 = 0.001$
 $n = 25$

Source : 50grp25
 Output : 50grp25.gph

%off



replicates

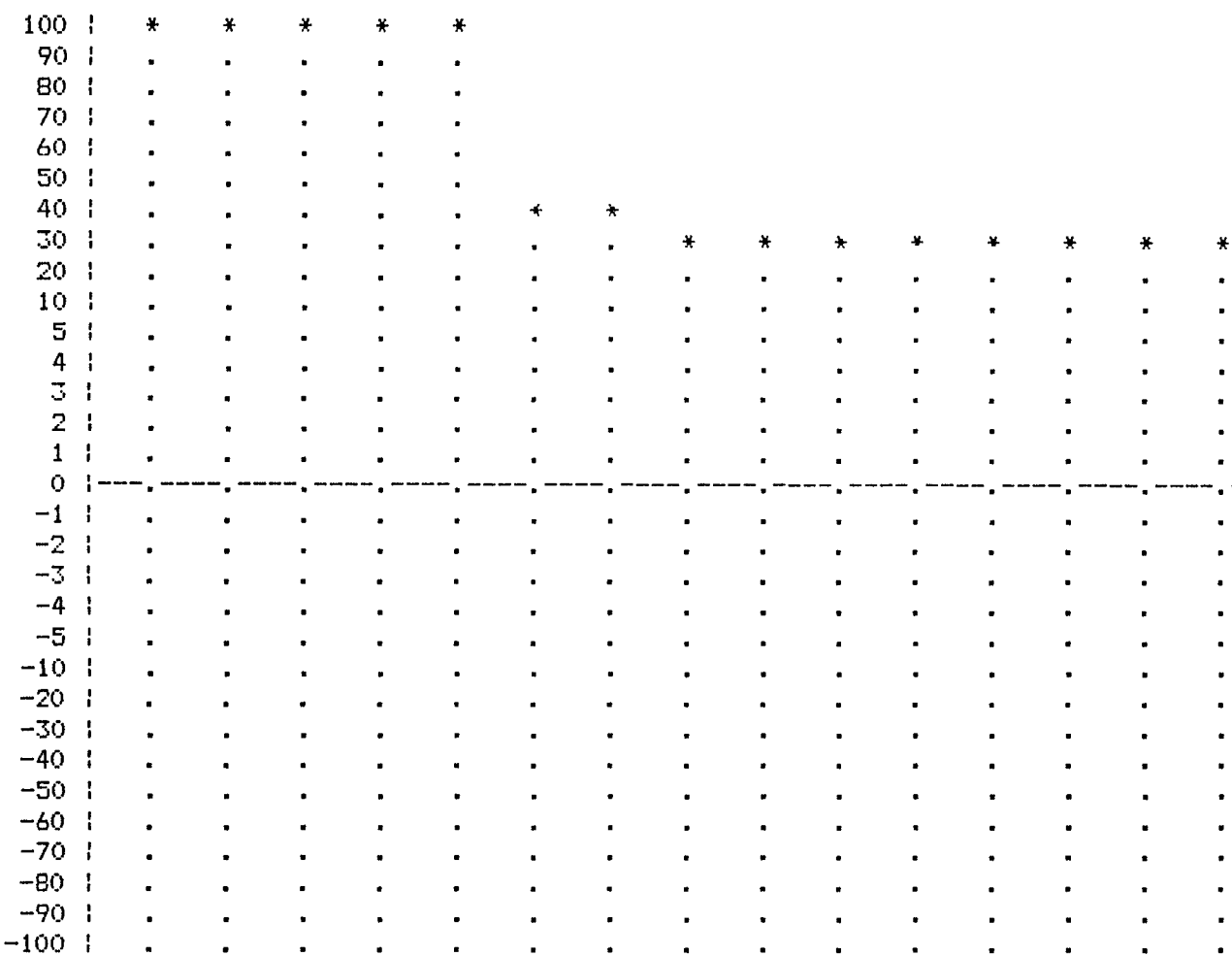
rep	b1		b0		tsum	te
	Mean	Std	Mean	Std	Mean	Mean
1	0.00088919	0.00105804	874.91	1593.78	8082.6	695.2
2	0.00063812	0.00068171	714.01	1106.09	7508.7	635.3
3	0.00063650	0.00049381	512.53	1234.86	7859.6	672.7
4	0.00059700	0.00043540	271.57	391.72	7518.8	641.8
5	0.00060762	0.00043377	309.43	651.91	7801.7	668.3
10	0.00058275	0.00028235	136.10	172.63	7647.1	654.4
20	0.00060125	0.00023832	127.50	229.72	7749.9	664.0
30	0.00060250	0.00019558	84.39	27.36	7703.6	660.1
40	0.00060875	0.00013206	78.88	15.10	7690.3	659.4
50	0.00061950	0.00014127	78.30	15.19	7631.4	654.9
60	0.00061550	0.00012939	77.96	16.80	7730.7	663.6
70	0.00059275	0.00011082	79.76	13.49	7708.3	659.5
80	0.00060900	0.00012016	78.44	13.92	7676.5	658.1
90	0.00061025	0.00012509	79.32	18.60	7660.4	656.6
100	0.00061200	0.00009785	76.84	11.30	7720.3	662.2

Summary Graph of % Mean B_0 Estimate differs from Γ_0

$\Gamma_0 = 50$ $\Phi_1 = 0.001$
 $n = 30$

Source : 50grp30
 Output : 50grp30.gph

%off



replicates

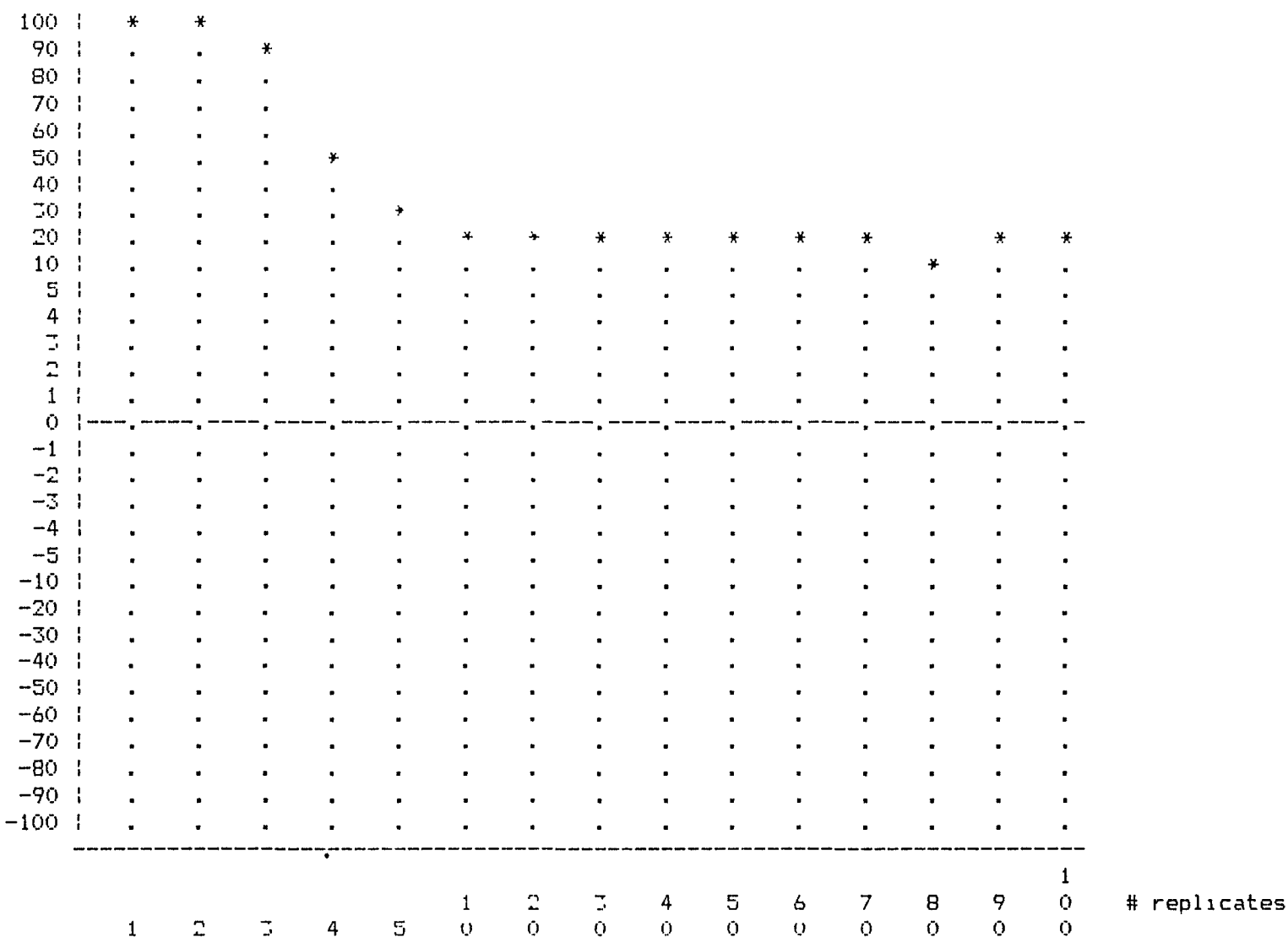
rep	b1		b0		tsum	te
	Mean	Std	Mean	Std	Mean	Mean
1	0.00081481	0.00076650	776.62	1784.15	12200.0	914.6
2	0.00072681	0.00051628	415.19	1050.56	11271.9	837.2
3	0.00073987	0.00044300	183.05	473.13	11849.0	883.8
4	0.00070494	0.00036349	123.14	164.59	11310.3	838.3
5	0.00072750	0.00039301	113.60	141.62	11788.2	881.4
10	0.00072394	0.00020323	70.04	18.51	11518.4	857.6
20	0.00071281	0.00018060	69.73	19.10	11665.0	867.2
30	0.00073825	0.00012294	65.02	8.55	11602.5	865.2
40	0.00075487	0.00008714	63.12	5.14	11595.0	866.9
50	0.00074812	0.00009937	64.27	6.80	11501.6	858.4
60	0.00073762	0.00007642	63.97	5.15	11652.1	869.1
70	0.00073425	0.00007466	64.39	5.05	11605.0	864.6
80	0.00074825	0.00008133	63.67	5.44	11566.8	863.4
90	0.00075062	0.00007675	63.57	5.33	11540.6	861.8
100	0.00074812	0.00006161	63.08	4.19	11640.6	870.0

' Summary Graph of % Mean B_0 Estimate differs from Γ_0

$\Gamma_0 = 50$ $\Phi_1 = 0.001$
 $n = 35$

Source : 50grp35
 Output : 50grp35.gph

%off



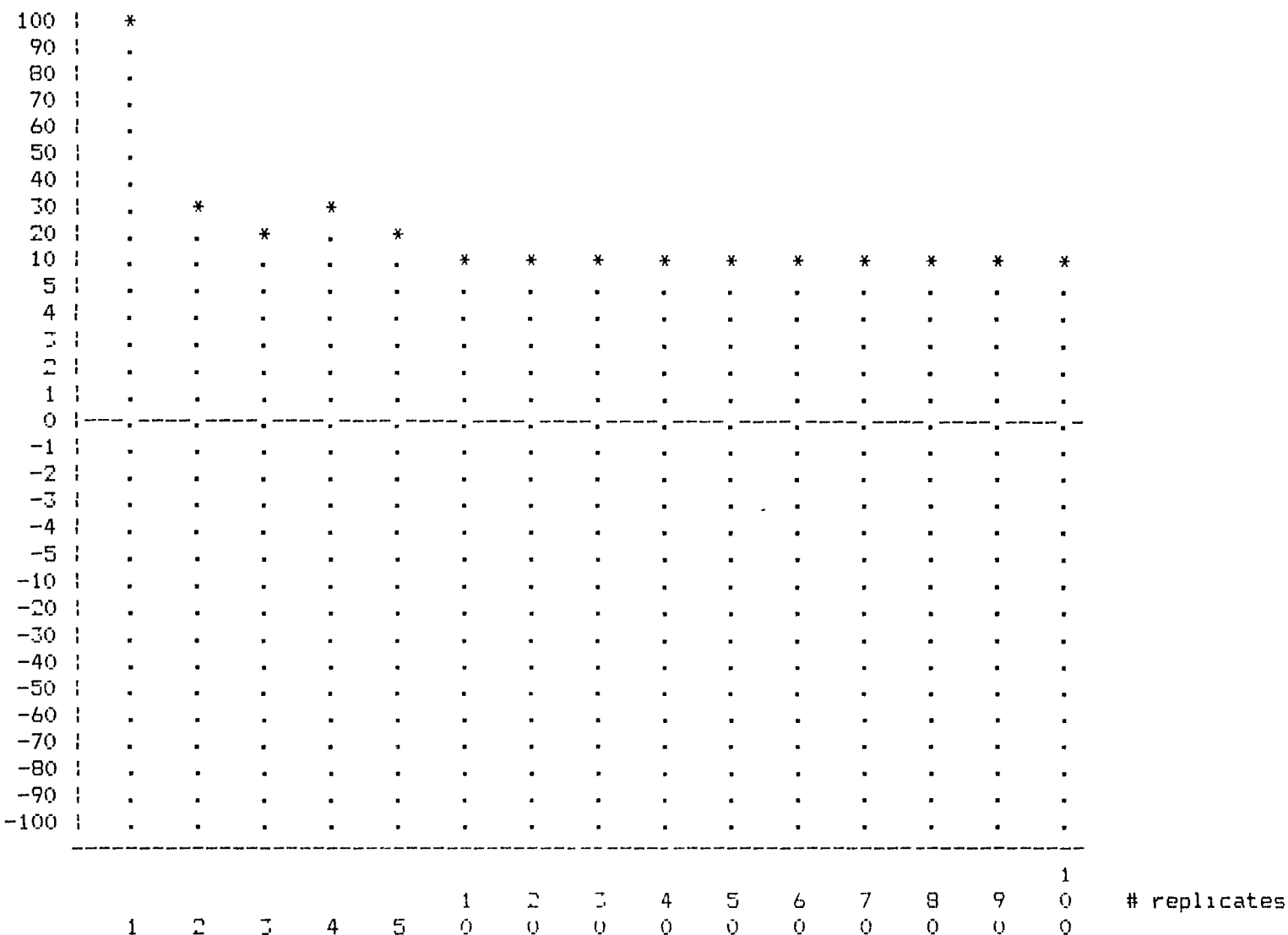
rep	b1		b0		tsum	te
	Mean	Std	Mean	Std	Mean	Mean
1	0.00075247	0.00057464	757.89	2031.99	17490.5	1174.6
2	0.00082453	0.00037238	167.21	464.29	16211.0	1094.5
3	0.00076544	0.00037020	95.97	131.62	17009.6	1137.3
4	0.00076544	0.00026849	73.56	35.14	16215.6	1083.2
5	0.00082703	0.00024253	64.30	31.63	16992.1	1154.2
10	0.00080875	0.00013094	60.49	7.61	16551.0	1111.7
20	0.00079331	0.00012291	60.63	7.05	16739.0	1121.8
30	0.00082400	0.00009308	58.58	5.12	16680.4	1125.0
40	0.00082987	0.00008022	58.10	4.18	16679.2	1126.8
50	0.00082878	0.00007027	58.46	3.55	16521.5	1113.5
60	0.00082331	0.00005481	58.04	3.00	16752.3	1130.7
70	0.00081356	0.00005914	58.77	3.12	16669.8	1120.7
80	0.00084131	0.00005773	57.39	3.01	16646.9	1127.2
90	0.00083462	0.00006276	57.86	3.09	16601.6	1121.3
100	0.00082681	0.00004884	57.79	2.43	16747.9	1131.1

Summary Graph of % Mean B_0 Estimate differs from Γ_0

$\Gamma_0 = 50$ $\Phi_1 = 0.001$
 $n = 40$

Source : 50grp40
 Output : 50grp40.gph

%off

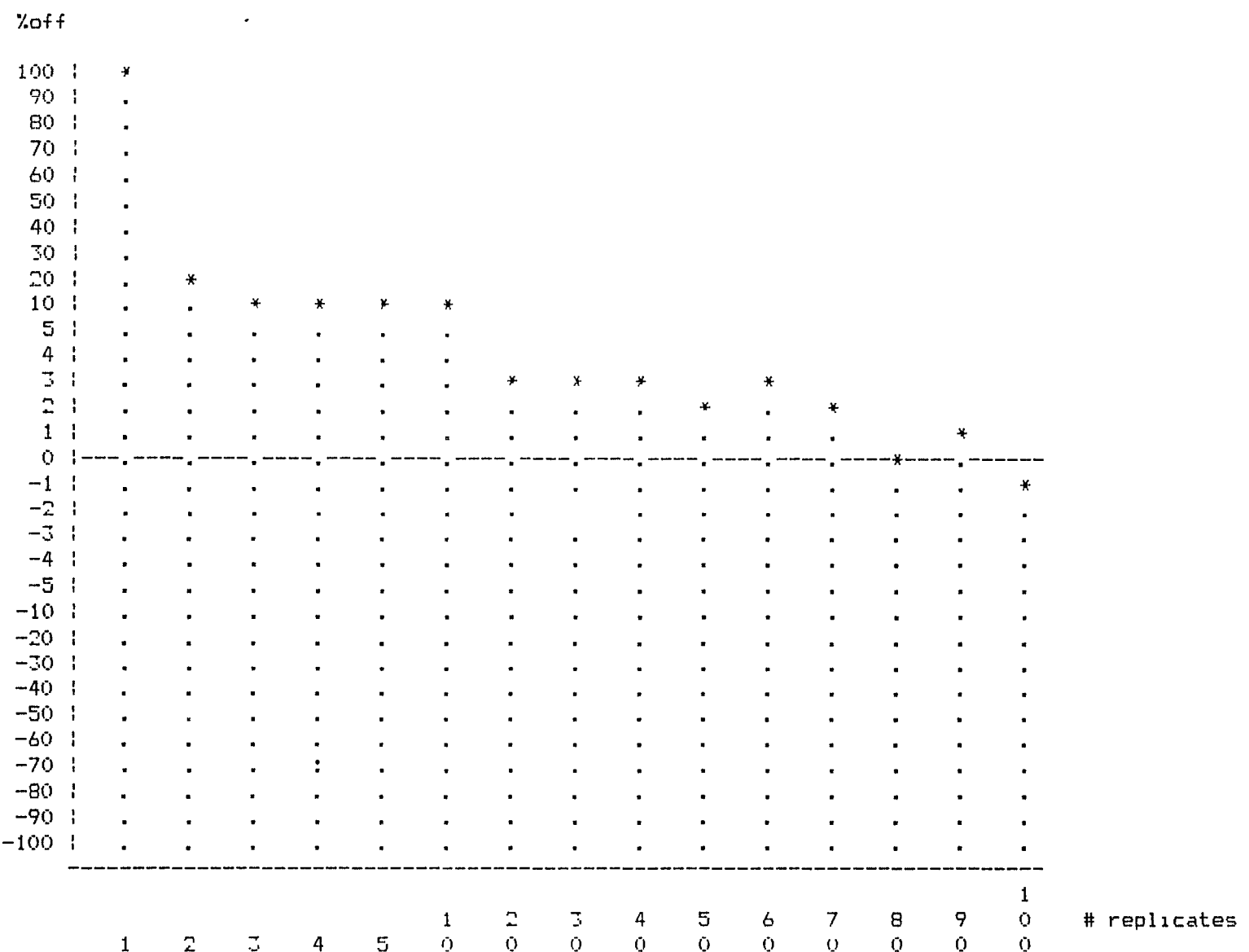


rep	b1		b0		tsum	te
	Mean	Std	Mean	Std	Mean	Mean
1	0.00070886	0.00045880	294.65	934.83	24372.6	1546.5
2	0.00085534	0.00026016	66.38	37.39	22710.3	1432.7
3	0.00085205	0.00026871	59.83	13.81	23746.8	1502.6
4	0.00084919	0.00020110	66.37	48.01	22587.3	1423.1
5	0.00086344	0.00020782	58.69	15.18	23848.3	1538.6
10	0.00088722	0.00013074	55.88	5.04	23144.3	1472.2
20	0.00088119	0.00009452	55.40	4.45	23405.5	1492.6
30	0.00089909	0.00007681	54.53	3.19	23350.8	1494.7
40	0.00088487	0.00006745	55.05	2.93	23330.0	1484.0
50	0.00090106	0.00007794	54.80	3.12	23140.6	1489.3
60	0.00089547	0.00006792	54.46	2.98	23462.6	1507.2
70	0.00089244	0.00006091	54.73	2.77	23305.9	1486.4
80	0.00091595	0.00006975	53.80	2.51	23334.3	1504.5
90	0.00090512	0.00005764	54.29	2.33	23242.3	1487.5
100	0.00090487	0.00005504	53.97	2.13	23447.5	1505.2

Summary Graph of % Mean B_0 Estimate differs from Γ_0

$\Gamma_0 = 50$ $\Phi_1 = 0.001$
 $n = 45$

Source : 50grp45
 Output : 50grp45.gph



rep	b1		b0		tsum	te
	Mean	Std	Mean	Std	Mean	Mean
1	0.00075474	0.00031450	349.03	1699.32	34147.7	2366.4
2	0.00090554	0.00025409	60.12	23.73	31523.4	2041.8
3	0.00086647	0.00017928	56.67	7.72	32763.3	2062.4
4	0.00089416	0.00015627	57.44	10.74	31426.4	2090.1
5	0.00088595	0.00015137	55.04	7.29	33673.8	2418.0
10	0.00094281	0.00010921	53.23	4.17	32342.1	2197.8
20	0.00098434	0.00008700	51.31	3.50	33137.9	2440.2
30	0.00097758	0.00008922	51.57	3.21	32898.0	2334.7
40	0.00097745	0.00008508	51.65	2.98	32668.8	2254.3
50	0.00099222	0.00012149	51.15	3.24	33812.6	2970.7
60	0.00096982	0.00008830	51.28	3.19	33846.9	2786.7
70	0.00100105	0.00010385	51.00	3.44	33017.8	2461.0
80	0.00103154	0.00010391	49.80	3.08	33732.1	2798.5
90	0.00101009	0.00008627	50.65	2.90	32961.1	2453.3
100	0.00103441	0.00008439	49.63	2.70	33608.4	2657.1

Summary Graph of % Mean B_0 Estimate differs from Γ_0

$\Gamma_0 = 100$ $\Phi_1 = 0.001$
 $n = 10$

Source : 100grp10
 Output : 100grp10.gph

%off

100	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*
90
80
70
60
50
40
30
20
10
5
4
3
2
1
0	-----														
-1
-2
-3
-4
-5
-10
-20
-30
-40
-50
-60
-70
-80
-90
-100

						1	2	3	4	5	6	7	8	9	1	
						0	0	0	0	0	0	0	0	0	0	# replicates
1	2	3	4	5	0	0	0	0	0	0	0	0	0	0	0	

rep	b1		b0		tsum	te
	Mean	Std	Mean	Std	Mean	Mean
1	0.00191850	0.00379749	1591.31	1327.52	557.6	102.9
2	0.00092650	0.00214355	1570.82	943.85	610.9	110.7
3	0.00082150	0.00178830	1628.78	946.84	535.1	100.6
4	0.00065400	0.00175157	1527.30	811.41	581.8	107.7
5	0.00021400	0.00085738	1836.92	560.31	563.8	102.8
10	0.00009000	0.00022978	1850.92	410.50	577.7	105.7
20	0.00005000	0.00000000	1949.96	148.06	562.8	103.4
30	0.00005000	0.00000000	1951.87	115.25	556.0	103.1
40	0.00005000	0.00000000	1938.74	95.61	564.4	103.7
50	0.00005000	0.00000000	1958.14	100.84	555.3	102.7
60	0.00005000	0.00000000	1935.73	80.19	560.6	103.8
70	0.00005000	0.00000000	1929.10	68.68	563.9	104.1
80	0.00005000	0.00000000	1931.84	69.27	562.3	103.9
90	0.00005000	0.00000000	1920.90	77.43	566.0	104.6
100	0.00005000	0.00000000	1938.02	65.75	556.4	103.6

Summary Graph of % Mean B_0 Est1 differs from Γ_0

$\Gamma_0 = 100$ $\Phi_1 = 0.001$
 $n = 20$

Source : 100grp20
 Output : 100grp20.gph

off

00	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*
90
80
70
60
50
40
30
20
10
5
4
3
2
1
0
-1
-2
-3
-4
-5
-10
-20
-30
-40
-50
-60
-70
-80
-90
100

						1	2	3	4	5	6	7	8	9	10	# replicates
1	2	3	4	5	0	0	0	0	0	0	0	0	0	0	0	

rep	b1		b0		tsum	te
	Mean	Std	Mean	Std	Mean	Mean
1	0.00137475	0.00226914	1784.37	1646.17	2216.1	218.0
2	0.00069475	0.00137612	2250.08	1849.09	2338.9	223.1
3	0.00088600	0.00117673	1273.54	1239.87	2147.4	214.9
4	0.00053250	0.00072407	1355.50	1227.01	2298.1	225.8
5	0.00040850	0.00081432	1415.26	957.32	2222.7	219.7
10	0.00032150	0.00051798	1376.00	802.54	2265.7	222.8
20	0.00023600	0.00036387	1414.19	740.55	2213.2	218.8
30	0.00010800	0.00016624	1616.19	579.77	2202.1	216.7
40	0.00010600	0.00013588	1568.21	592.49	2222.7	218.7
50	0.00012800	0.00019828	1479.32	637.62	2213.0	219.2
60	0.00011400	0.00013676	1530.36	624.36	2219.6	218.8
70	0.00007000	0.00006928	1697.30	426.84	2232.7	219.5
80	0.00009000	0.00010954	1625.66	521.92	2225.0	219.4
90	0.00005600	0.00003105	1781.94	266.19	2233.1	219.2
100	0.00008200	0.00010284	1664.55	485.59	2211.7	218.4

Summary Graph of % Mean B_0 Estimate differs from Γ_{B_0}

$\Gamma_{B_0} = 100$ $\Phi_1 = 0.001$
 $n = 30$

Source : 100grp30
 Output : 100grp30.gph

%off

100	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*
90
80
70
60
50
40
30
20
10
5
4
3
2
1
0	-----														
-1
-2
-3
-4
-5
-10
-20
-30
-40
-50
-60
-70
-80
-90
-100

1
 # replicates

1 2 3 4 5 1 2 3 4 5 6 7 8 9 0

rep	b1		b0		tsum	te
	Mean	Std	Mean	Std	Mean	Mean
1	0.00117525	0.00159097	2846.65	3694.71	5069.6	344.0
2	0.00054825	0.00081416	1833.67	1991.62	5313.3	357.8
3	0.00071025	0.00074835	1304.80	1668.29	4974.6	340.3
4	0.00055625	0.00064142	1362.06	1909.29	5275.8	356.9
5	0.00042950	0.00041215	981.76	1146.18	5107.9	346.5
10	0.00041975	0.00039233	939.69	1184.85	5199.1	353.0
20	0.00059450	0.00038280	419.69	581.14	5117.1	352.7
30	0.00045300	0.00030223	454.98	645.05	5072.1	346.8
40	0.00042050	0.00026738	475.76	556.35	5114.7	349.4
50	0.00044300	0.00025510	419.01	502.51	5102.3	348.8
60	0.00038400	0.00019861	367.99	370.57	5097.1	347.4
70	0.00042000	0.00016439	285.55	244.18	5135.8	350.9
80	0.00042300	0.00018384	326.33	362.88	5124.1	350.0
90	0.00035450	0.00019517	415.65	415.05	5126.1	349.0
100	0.00042400	0.00021148	374.09	428.66	5095.9	348.3

Summary Graph of % Mean B_0 Estimate differs from $\Gamma\gamma\alpha_0$

$\Gamma\gamma\alpha_0 = 100$ $\Phi\eta_1 = 0.001$
 $n = 40$

Source : 100grp40
 Output : 100grp40.gph

%off

100	*	*	*	*	*										
90	*									
80									
70									
60									
50		*		*			*		
40	*	*	.	*	.	*	*	.	*
30
20
10
5
4
3
2
1
0															
-1
-2
-3
-4
-5
-10
-20
-30
-40
-50
-60
-70
-80
-90
-100

1

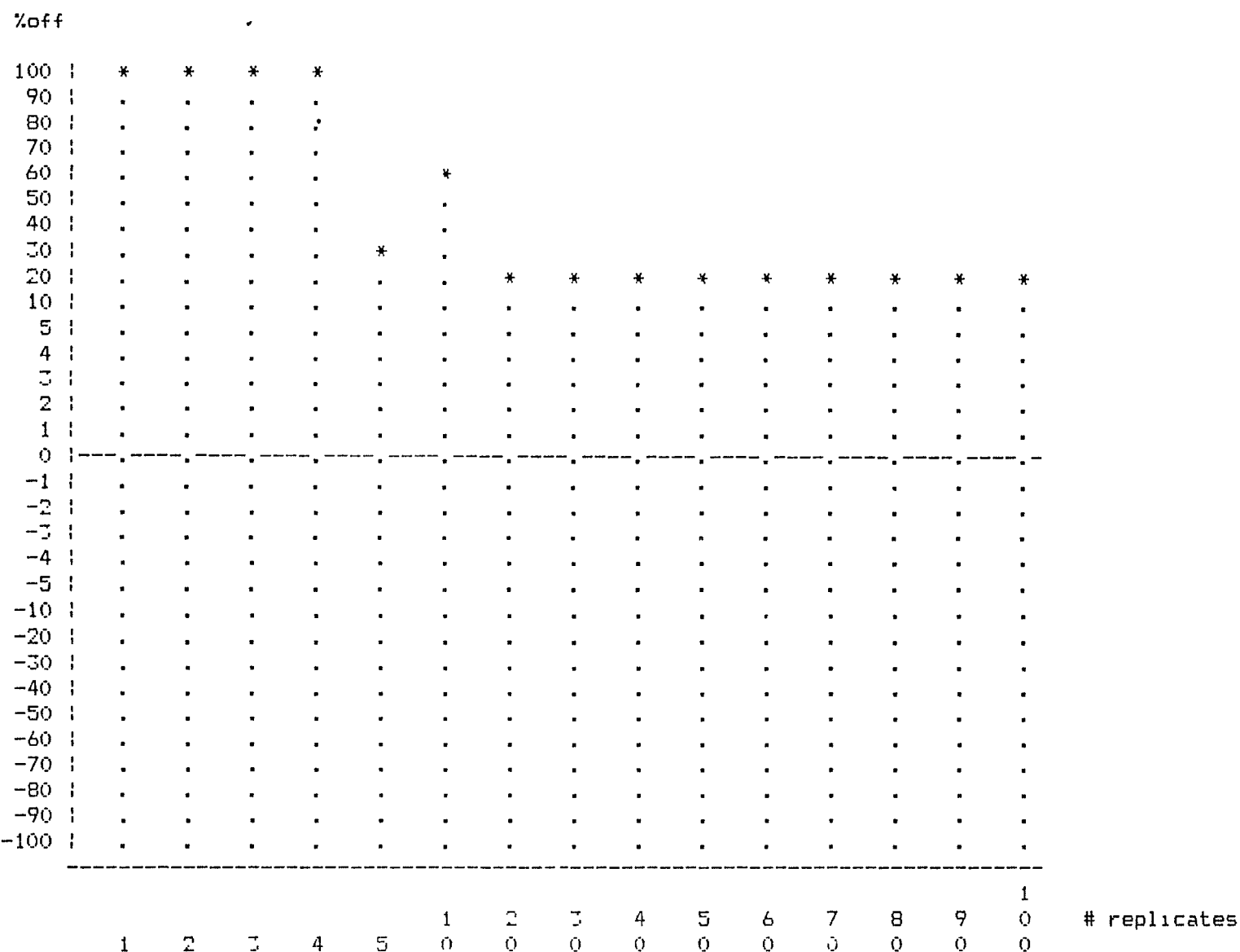
replicates

rep	b1		b0		tsum	te
	Mean	Std	Mean	Std	Mean	Mean
1	0.00093056	0.00100241	2846.53	4418.66	9329.6	492.9
2	0.00075312	0.00073663	1288.54	2209.80	9720.3	514.4
3	0.00078575	0.00059107	791.97	1331.60	9178.3	489.6
4	0.00067150	0.00054180	1192.69	2701.43	9652.1	507.9
5	0.00062550	0.00041662	600.99	1009.36	9366.9	492.6
10	0.00064612	0.00029787	186.98	116.46	9553.4	505.3
20	0.00073800	0.00021458	139.89	43.47	9453.4	503.9
30	0.00071300	0.00016591	141.31	32.89	9353.7	497.0
40	0.00068525	0.00016998	148.18	45.29	9421.5	499.7
50	0.00071125	0.00014503	139.98	34.70	9404.2	499.8
60	0.00066800	0.00012104	145.88	23.21	9377.3	496.3
70	0.00067400	0.00011942	143.33	20.92	9453.9	500.9
80	0.00069025	0.00011571	140.95	24.55	9433.4	500.4
90	0.00066500	0.00013231	146.57	24.54	9421.2	498.6
100	0.00067750	0.00011161	143.46	21.69	9381.7	497.1

Summary Graph of % Mean B_0 Estimate differs from Γ_{B_0}

$\Gamma_{B_0} = 100$ $\Phi_{B_1} = 0.001$
 $n = 50$

Source : 100grp50
 Output : 100grp50.gph



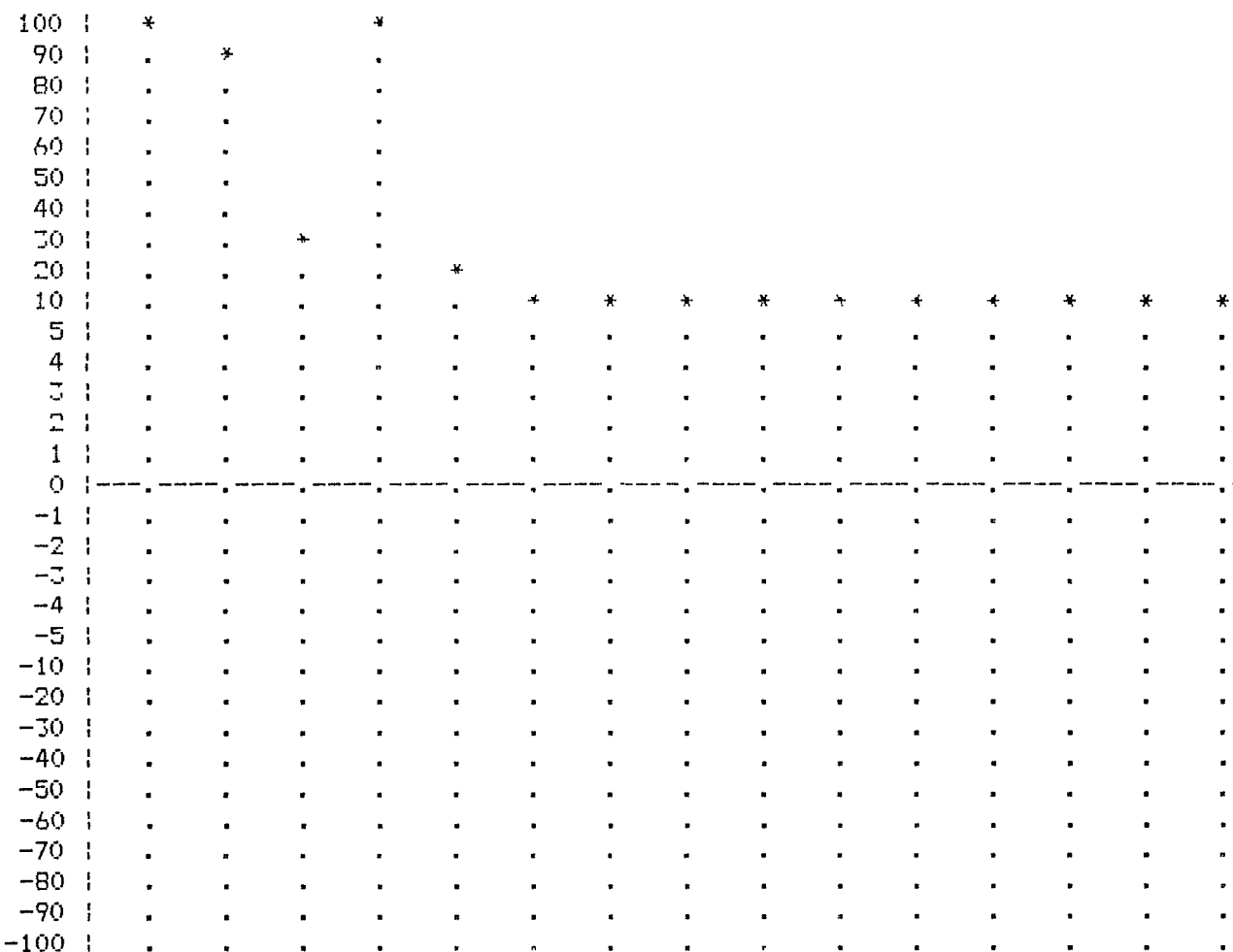
rep	b1		b0		tsum	te
	Mean	Std	Mean	Std	Mean	Mean
1	0.00086156	0.00062904	987.87	2807.85	15242.3	674.8
2	0.00070362	0.00048661	560.43	1864.33	15747.2	682.2
3	0.00081587	0.00048236	506.48	1277.25	14983.3	657.5
4	0.00080425	0.00041779	925.63	3490.78	15687.1	688.0
5	0.00084712	0.00028776	134.55	70.32	15283.7	676.5
10	0.00078012	0.00025691	159.40	196.58	15562.1	682.7
20	0.00080812	0.00017277	123.77	26.32	15425.0	679.0
30	0.00083275	0.00010148	117.93	11.56	15274.8	673.8
40	0.00081887	0.00010578	119.22	13.22	15371.1	677.3
50	0.00081225	0.00010320	120.00	11.77	15340.9	675.0
60	0.00081112	0.00008857	120.08	10.13	15289.6	672.4
70	0.00081150	0.00007810	118.99	8.43	15407.9	678.1
80	0.00080850	0.00007010	119.40	8.35	15386.3	676.8
90	0.00080212	0.00007336	120.48	8.16	15345.0	674.2
100	0.00079787	0.00008087	121.67	10.03	15283.5	670.9

' Summary Graph of % Mean E_0 Estimate differs from Γ_{α_j}

$\Gamma_{\alpha_0} = 100$ $\Phi_1 = 0.001$
 $n = 60$

Source : 100grp60
 Output : 100grp60.gpn

%off



																1
																# replicates
1	2	3	4	5	1	2	3	4	5	6	7	8	9	0	0	

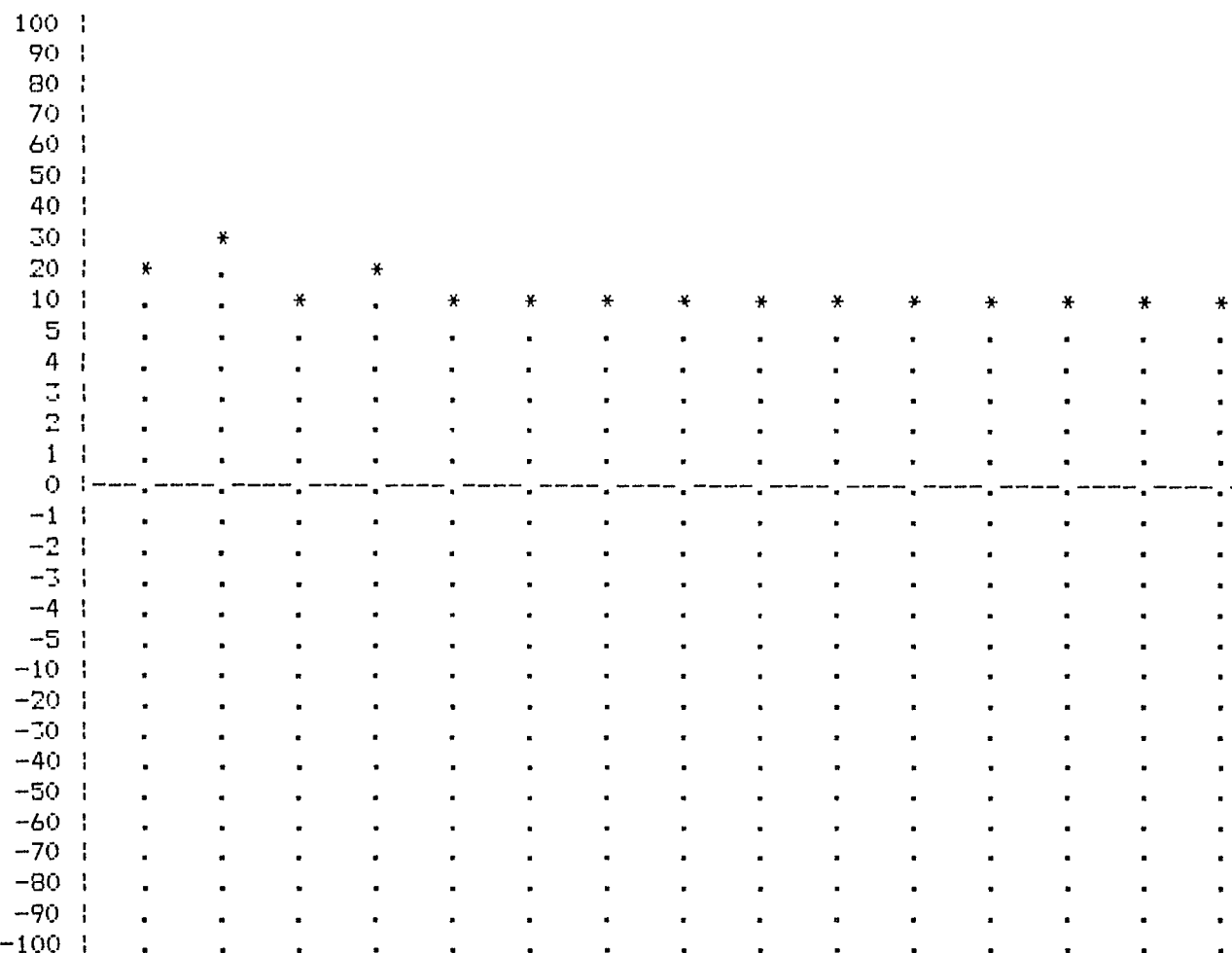
rep	b1		b0		tsum	te
	Mean	Std	Mean	Std	Mean	Mean
1	0.00086331	0.00047275	758.97	2445.93	23107.9	885.4
2	0.00081491	0.00040771	191.47	266.53	23764.0	900.5
3	0.00090537	0.00032583	132.33	81.17	22676.5	870.7
4	0.00083819	0.00032578	396.95	1586.67	23705.0	904.1
5	0.00091250	0.00021141	115.20	34.11	23182.6	894.3
10	0.00086587	0.00015624	114.12	17.60	23550.6	901.8
20	0.00086625	0.00010534	112.82	10.63	23350.2	892.2
30	0.00089131	0.00007702	110.48	7.36	23148.6	887.7
40	0.00087894	0.00006928	111.07	6.67	23281.4	891.5
50	0.00088731	0.00006831	110.34	6.07	23235.2	890.7
60	0.00088825	0.00006427	110.48	5.69	23156.0	887.3
70	0.00088269	0.00006305	110.31	5.31	23342.4	894.7
80	0.00087803	0.00005979	110.93	6.00	23300.1	892.2
90	0.00087950	0.00005595	110.98	5.21	23227.5	889.0
100	0.00088269	0.00005215	110.97	4.62	23129.3	885.1

' Summary Graph of % Mean B_0 Estimate differs from Γ_{γ_0}

$\Gamma_{\gamma_0} = 100$ $\Phi_1 = 0.001$
 $n = 70$

Source : 100grp70
 Output : 100grp70.gph

%off



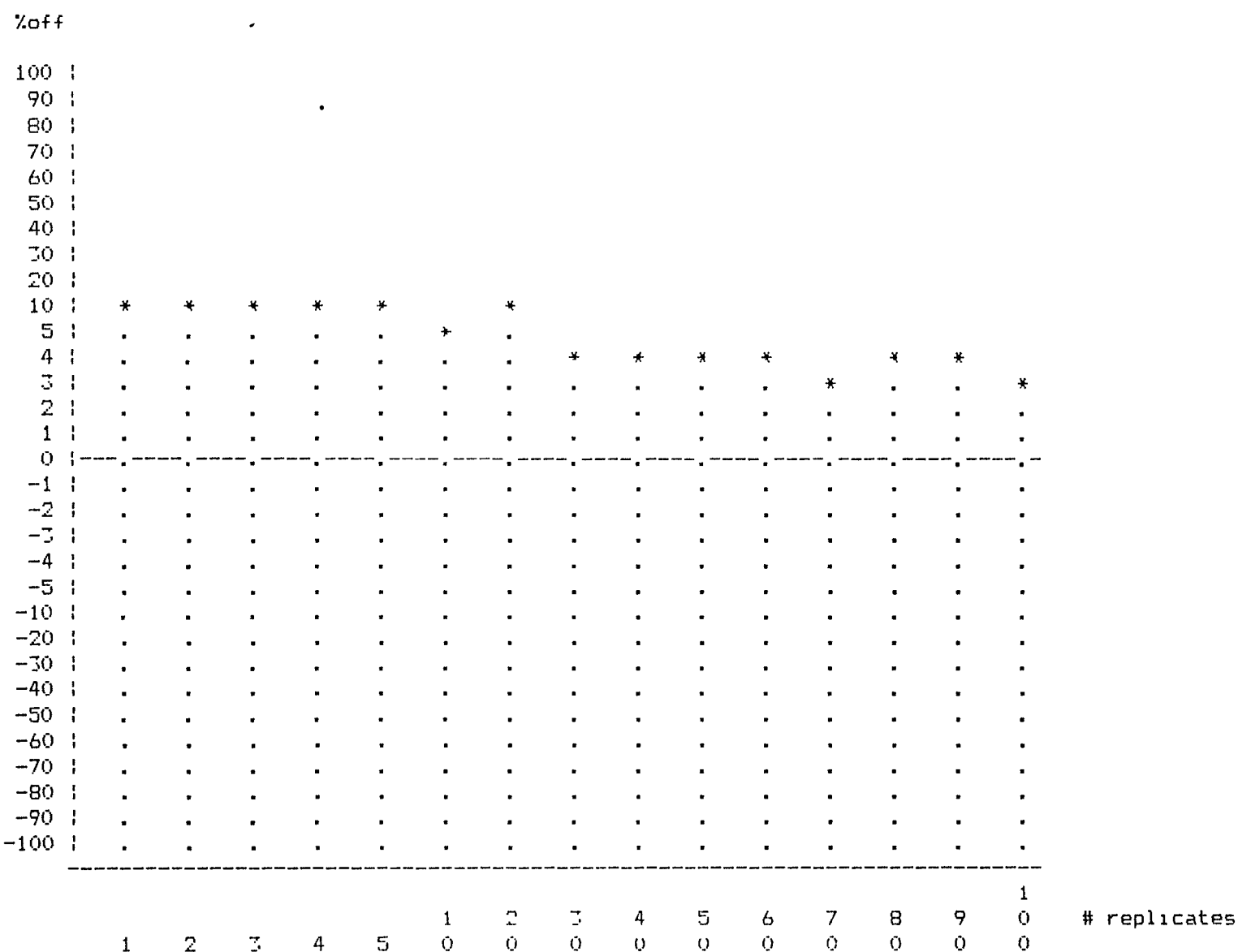
replicates

rep	b1		b0		tsum	te
	Mean	Std	Mean	Std	Mean	Mean
1	0.00096375	0.00034101	117.09	52.26	33483.8	1175.2
2	0.00085412	0.00030030	130.16	79.29	34166.4	1168.4
3	0.00094009	0.00023163	112.55	22.78	32847.6	1142.5
4	0.00087128	0.00021681	115.38	24.82	34233.9	1180.7
5	0.00090933	0.00013178	109.47	11.23	33516.0	1158.5
10	0.00089106	0.00009979	109.25	9.38	34011.0	1174.2
20	0.00089453	0.00007088	109.00	6.15	33690.1	1160.2
30	0.00092544	0.00005295	106.71	4.87	33470.5	1160.7
40	0.00092206	0.00004487	106.51	4.04	33644.4	1167.0
50	0.00092850	0.00005171	106.06	3.89	33603.8	1167.1
60	0.00091997	0.00004724	107.04	3.72	33460.4	1158.0
70	0.00091806	0.00004509	106.57	3.31	33731.8	1169.1
80	0.00091803	0.00004665	106.73	3.50	33663.7	1166.2
90	0.00091820	0.00003844	106.92	3.15	33550.5	1161.3
100	0.00092406	0.00003720	106.74	2.79	33410.9	1157.0

Summary Graph of % Mean B_0 Estimate differs from Γ_0

$\Gamma_0 = 100$ $\Phi_1 = 0.001$
 $n = 80$

Source : 100grp80
 Output : 100grp80.gph



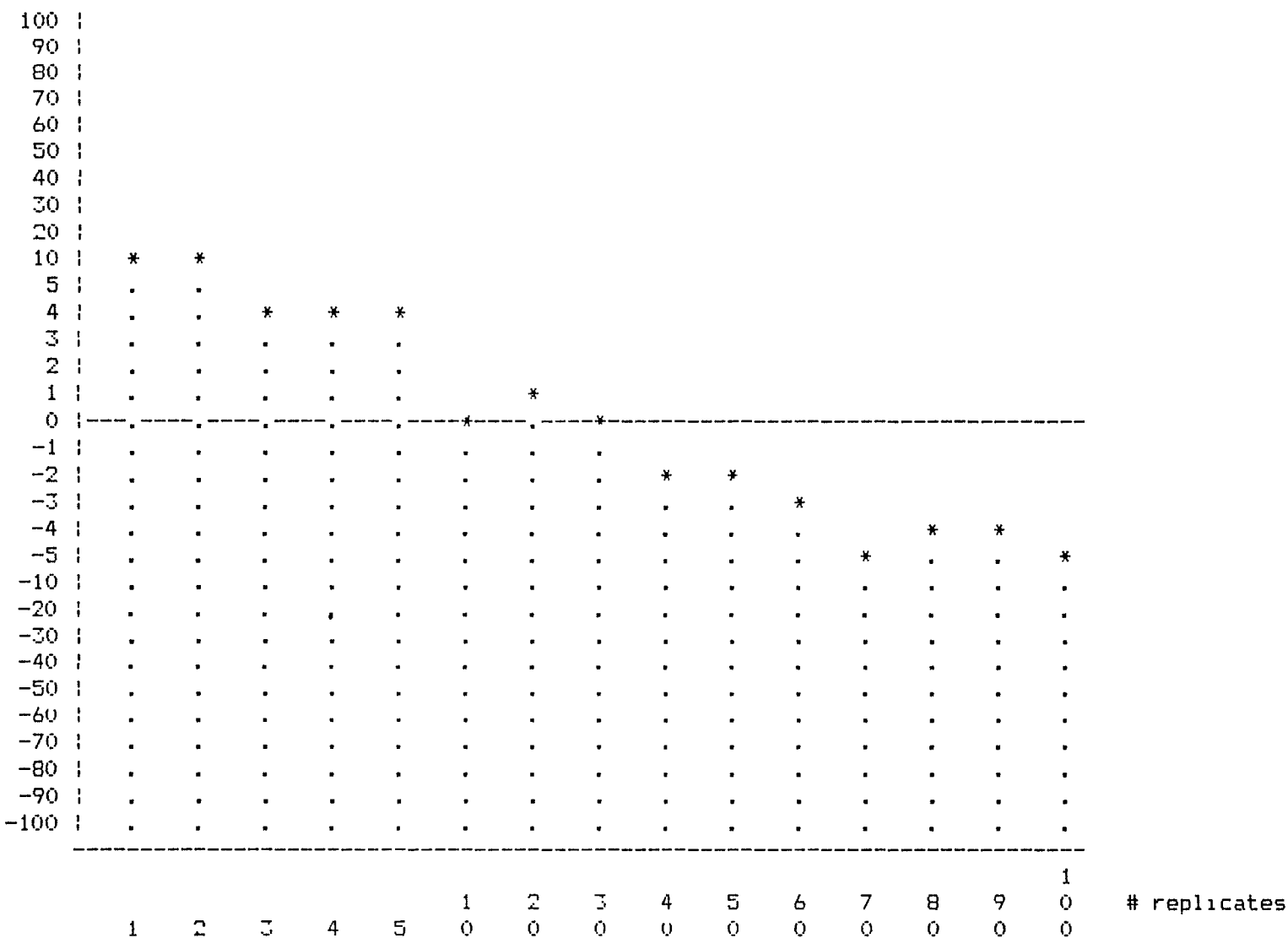
rep	b1		b0		tsum	te
	Mean	Std	Mean	Std	Mean	Mean
1	0.00093484	0.00025343	112.07	23.46	47245.8	1564.0
2	0.00087875	0.00021100	113.67	20.00	47760.7	1531.1
3	0.00096278	0.00014394	107.00	14.63	46267.4	1524.1
4	0.00090252	0.00015407	108.79	12.96	48022.1	1565.5
5	0.00092808	0.00009982	106.88	8.68	47088.7	1533.1
10	0.00093648	0.00007444	104.69	5.55	47792.4	1567.4
20	0.00092422	0.00005148	106.04	3.90	47203.0	1530.0
30	0.00095044	0.00004071	104.41	4.02	47042.0	1541.2
40	0.00095437	0.00003733	103.71	3.32	47317.4	1555.7
50	0.00095433	0.00003784	103.74	3.09	47271.5	1553.1
60	0.00095577	0.00004003	103.97	2.74	47012.7	1541.1
70	0.00095977	0.00003885	103.11	2.43	47440.4	1563.5
80	0.00095495	0.00003286	103.56	2.40	47325.8	1555.1
90	0.00095633	0.00003187	103.69	2.20	47152.3	1547.7
100	0.00096473	0.00002932	103.36	1.91	46973.0	1544.8

Summary Graph of % Mean B_0 Estimate differs from Γ_0

$\Gamma_0 = 100$ $\Phi_1 = 0.001$
 $n = 90$

Source : 100grp90
 Output : 100grp90.gph

%off



replicates

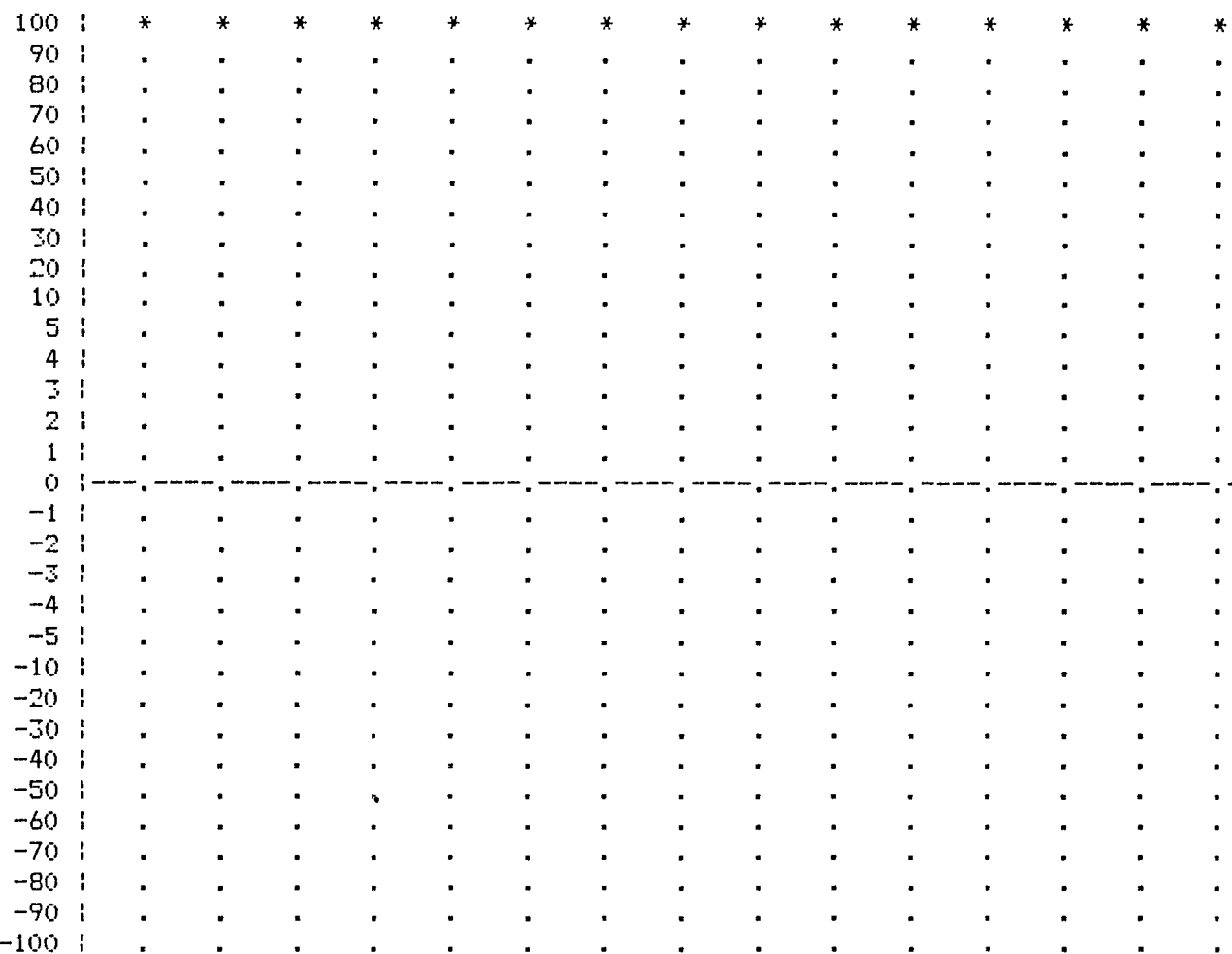
rep	b1		b0		tsum	te
	Mean	Std	Mean	Std	Mean	Mean
1	0.00091808	0.00021738	111.31	20.98	66033.2	2216.5
2	0.00089183	0.00012934	109.17	9.93	66001.0	2105.5
3	0.00098189	0.00011024	103.77	7.07	65569.9	2566.9
4	0.00093341	0.00013940	104.06	8.87	72326.9	4939.7
5	0.00097093	0.00008620	103.66	7.79	65956.1	2345.4
10	0.00099838	0.00009498	100.26	6.10	68715.0	3180.1
20	0.00099080	0.00008355	101.30	5.60	67455.7	3066.5
30	0.00101808	0.00010291	99.78	5.79	68463.0	3621.2
40	0.00103484	0.00009119	98.32	5.65	69379.4	3856.0
50	0.00105581	0.00010324	97.82	4.93	68365.9	3370.9
60	0.00108250	0.00011326	96.92	5.30	68344.1	3548.9
70	0.00109489	0.00011069	95.29	5.02	70645.0	4379.3
80	0.00110228	0.00009800	95.70	4.51	68873.6	3594.8
90	0.00109874	0.00009624	96.31	4.54	67845.2	3193.8
100	0.00110305	0.00009303	95.43	4.49	69391.0	4073.2

'Summary Graph of % Mean B_0 Est. (te differs from Γ_{γ_0})

$\Gamma_{\gamma_0} = 200$ $\Phi_1 = 0.001$
 $n = 20$

Source : 200grp10
 Output : 200grp10.gph

%off



replicates

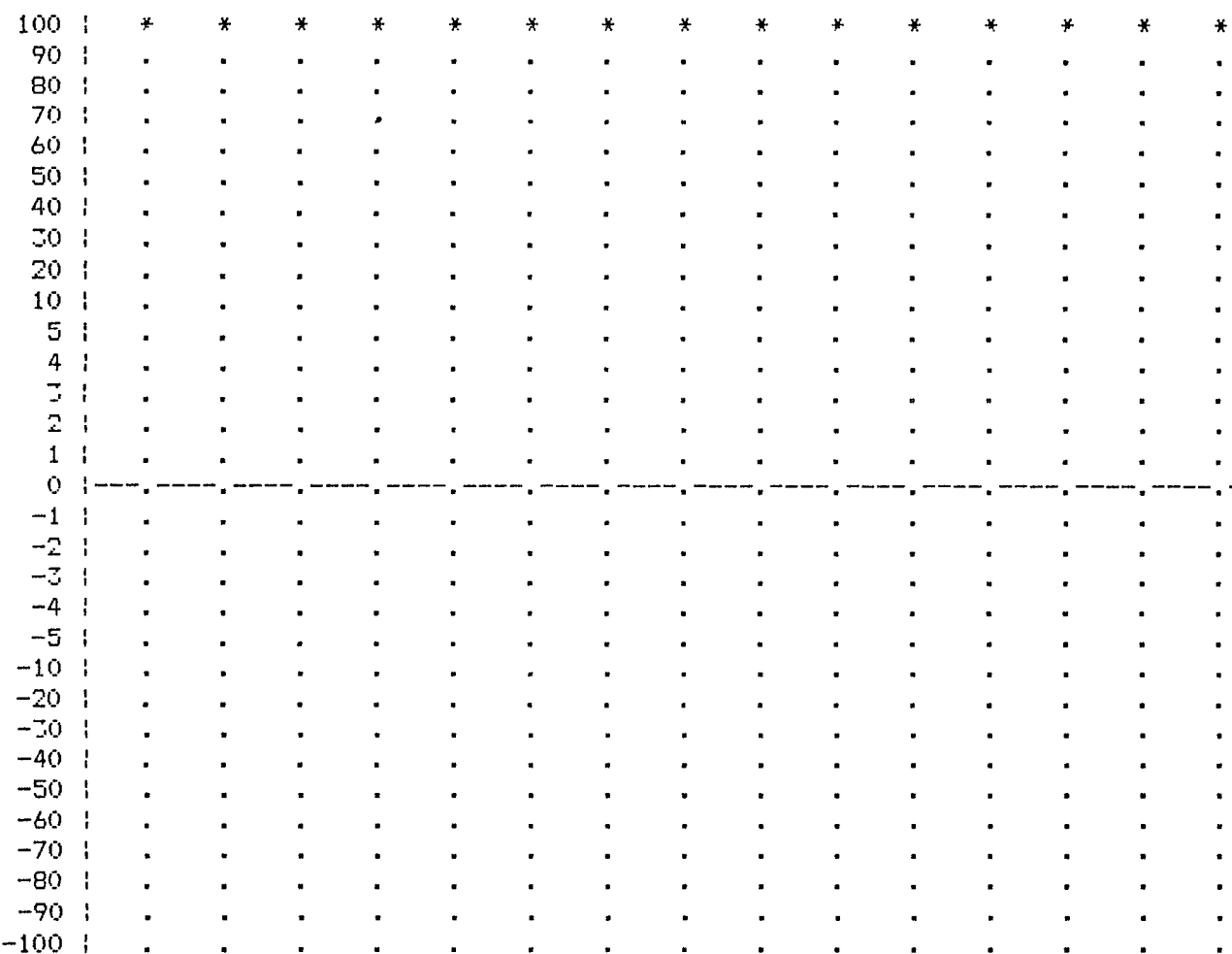
rep	b1		b0		tsum	te
	Mean	Std	Mean	Std	Mean	Mean
1	0.00139100	0.00297119	3209.80	2438.08	1119.1	105.4
2	0.00184900	0.00245699	2281.28	2142.36	1050.2	101.5
3	0.00078100	0.00145673	2923.14	1973.78	1098.8	105.5
4	0.00065550	0.00122881	2776.31	1704.98	1116.0	107.8
5	0.00030000	0.00087207	3420.15	1220.77	1097.0	105.0
10	0.00029600	0.00058007	3117.30	1465.89	1082.2	104.8
20	0.00010400	0.00030411	3710.15	749.30	1078.9	104.3
30	0.00016200	0.00033205	3429.13	1189.48	1068.5	104.0
40	0.00014800	0.00037013	3561.80	992.87	1074.7	104.3
50	0.00005000	0.00000000	3856.16	145.25	1076.6	104.1
60	0.00005800	0.00004400	3712.80	588.16	1080.4	104.8
70	0.00005000	0.00000000	3835.54	98.93	1081.7	104.6
80	0.00005000	0.00000000	3834.30	95.33	1079.7	104.7
90	0.00005000	0.00000000	3840.95	91.14	1077.1	104.5
100	0.00005000	0.00000000	3830.29	80.21	1086.0	104.8

' Summary Graph of % Mean B_0 Estimate differs from Γ_{γ_0}

$\Gamma_{\gamma_0} = 200$ $\Phi_{\gamma_1} = 0.001$
 $n = 40$

Source : 200grp20
 Output : 200grp20.gph

%off

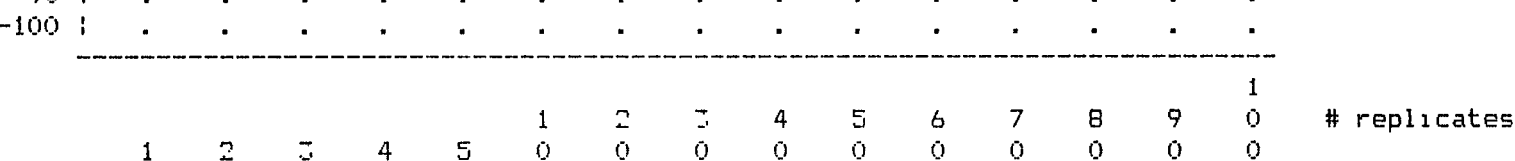


1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60 61 62 63 64 65 66 67 68 69 70 71 72 73 74 75 76 77 78 79 80 81 82 83 84 85 86 87 88 89 90 91 92 93 94 95 96 97 98 99 100
 # replicates

rep	b1		b0		tsum	te
	Mean	Std	Mean	Std	Mean	Mean
1	0.00109850	0.00153999	5692.62	7112.14	4411.5	219.6
2	0.00117300	0.00131250	2887.79	4240.24	4296.1	219.8
3	0.00057675	0.00068225	3580.74	4280.09	4378.3	218.4
4	0.00057675	0.00057641	2002.40	2691.00	4490.3	226.8
5	0.00052775	0.00068787	2450.99	2861.07	4393.1	220.6
10	0.00053550	0.00052393	1555.98	1701.62	4417.3	224.4
20	0.00046800	0.00039380	1348.32	1445.24	4361.4	220.9
30	0.00048050	0.00040888	1431.32	1536.65	4350.3	220.5
40	0.00040200	0.00032202	1332.51	1394.26	4345.4	220.1
50	0.00042400	0.00033634	1260.21	1373.72	4373.5	221.8
60	0.00031600	0.00026124	1504.56	1427.50	4378.3	221.1
70	0.00026800	0.00022600	1574.82	1388.66	4372.1	220.6
80	0.00029800	0.00020904	1241.97	1179.19	4371.3	220.7
90	0.00039000	0.00024166	989.20	1118.02	4375.1	221.8
100	0.00028400	0.00022504	1537.29	1405.63	4385.8	221.4


```
Source : 200grp30
Output : 200grp30.gph
```

•



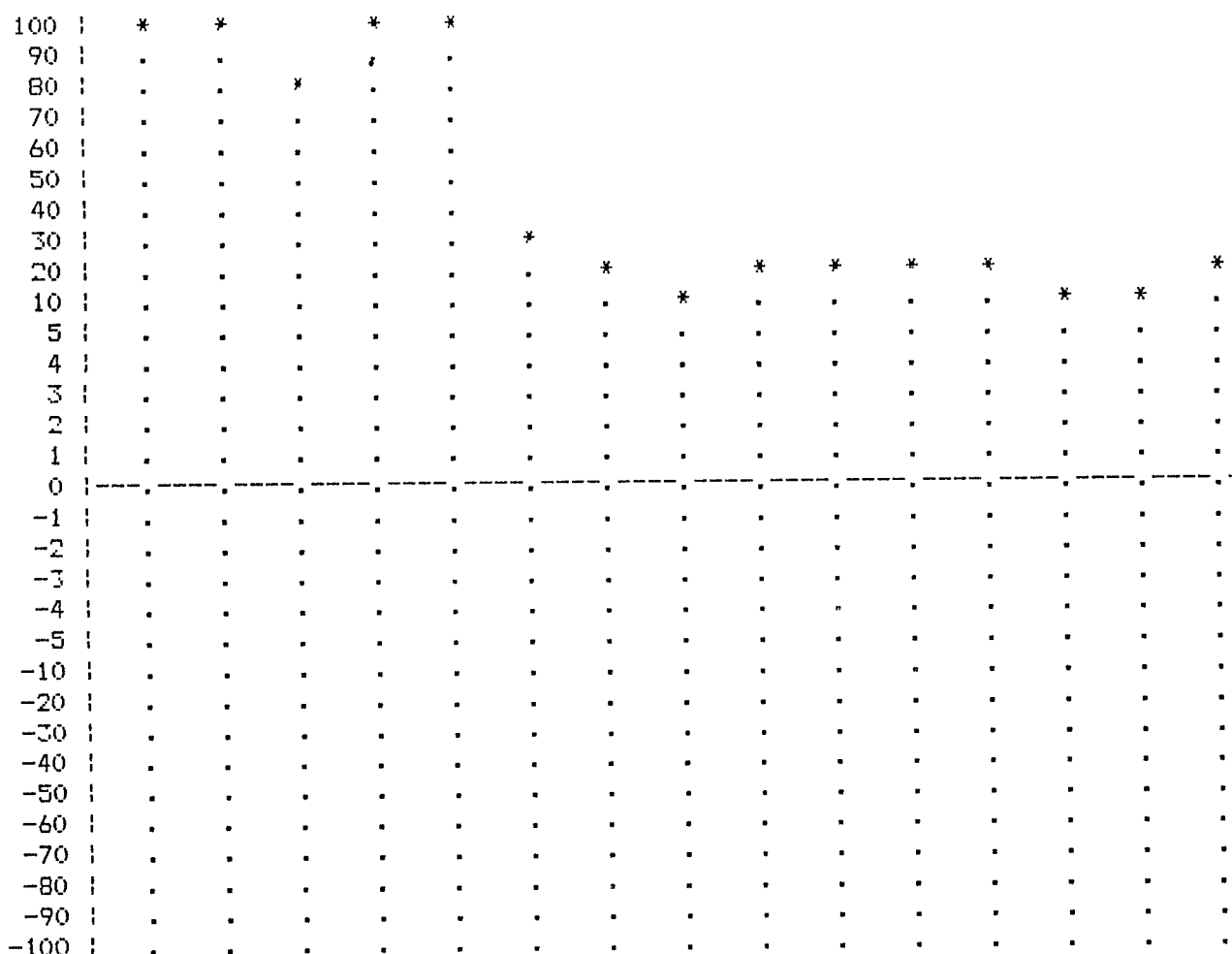
	b1		b0		tsum	te
rep	Mean	Std	Mean	Std	Mean	Mean
1	0.00077250	0.00079156	6097.46	10594.21	10159.8	348.5
2	0.00095837	0.00079194	1497.24	3119.54	10032.2	352.3
3	0.00082012	0.00060139	1990.64	4737.12	10079.1	351.3
4	0.00070562	0.00060629	1316.43	2463.58	10407.5	361.3
5	0.00073537	0.00050644	812.98	1293.35	10200.4	354.8
10	0.00067425	0.00033221	507.00	955.69	10244.8	355.7
20	0.00073600	0.00023981	297.76	121.17	10123.7	352.8
30	0.00072600	0.00021572	310.90	185.42	10113.2	352.1
40	0.00070975	0.00018375	292.37	81.86	10090.5	350.9
50	0.00074525	0.00018688	276.50	76.15	10173.4	354.7
60	0.00068025	0.00017086	312.08	147.25	10150.6	352.5
70	0.00066450	0.00015511	305.80	80.59	10143.2	352.0
80	0.00068950	0.00012166	285.91	44.67	10142.1	352.3
90	0.00071725	0.00014724	278.55	55.07	10171.9	354.1
100	0.00069200	0.00012424	284.89	47.47	10169.5	353.4

Summary Graph of % Mean B_0 Estimate differs from Γ_{a_1}

$\Gamma_{a_1} = 200$ $\Phi_1 = 0.001$
 $n = 80$

Source : 200grp40
 Output : 200grp40.gph

%off



replicates

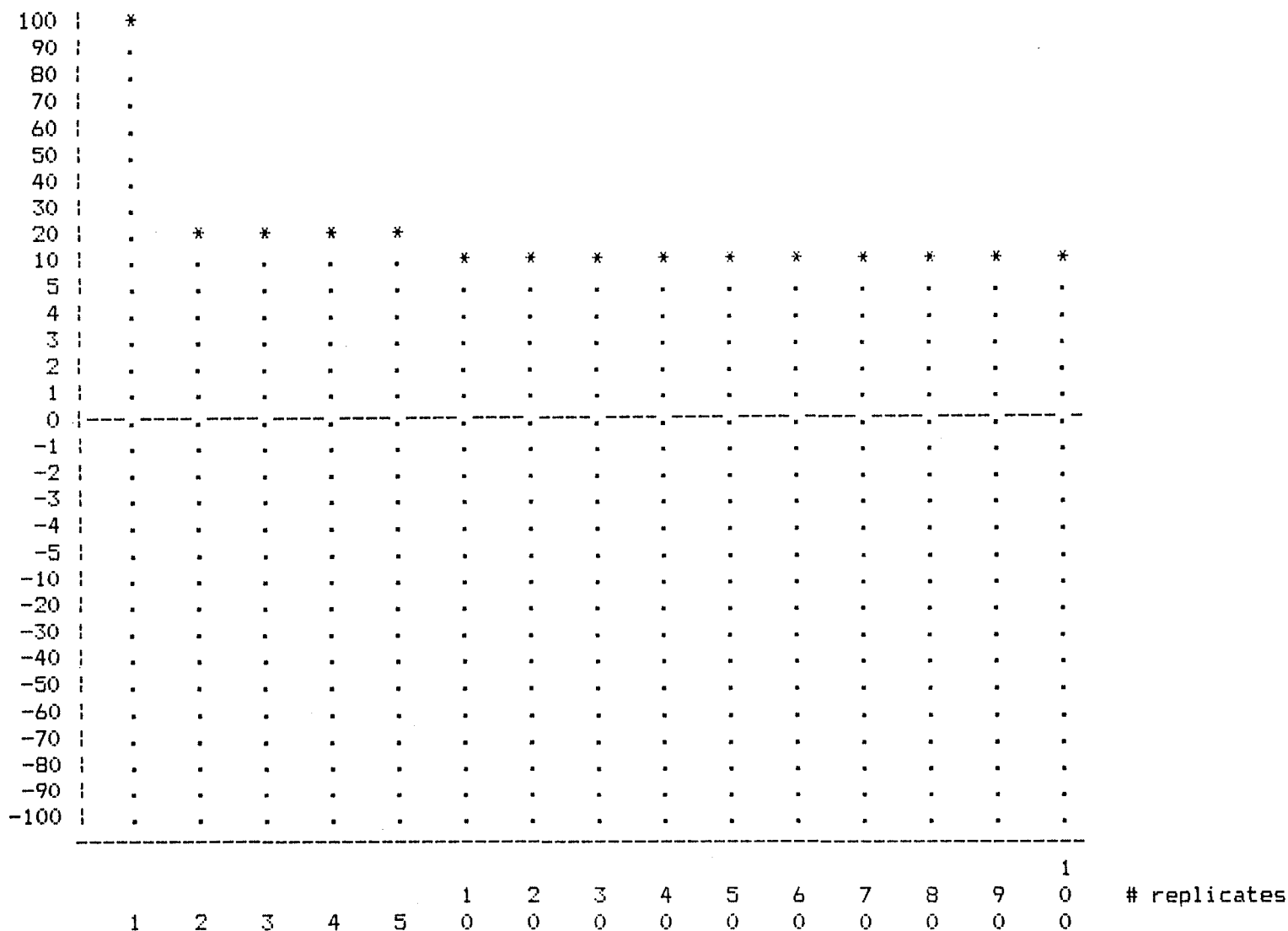
rep	b1		b0		tsum	te
	Mean	Std	Mean	Std	Mean	Mean
1	0.00080875	0.00067817	3285.72	6909.38	18675.1	498.5
2	0.00096106	0.00048345	498.87	1235.09	18733.7	509.6
3	0.00086825	0.00043701	359.43	450.47	18668.9	502.9
4	0.00070287	0.00036056	392.59	447.54	19167.4	509.6
5	0.00079412	0.00038853	407.50	623.56	18832.9	504.6
10	0.00079725	0.00021547	258.16	70.64	18932.5	507.8
20	0.00084375	0.00017337	241.03	49.15	18719.7	503.5
30	0.00087312	0.00014022	229.90	34.92	18719.8	504.9
40	0.00085450	0.00011074	232.66	26.93	18670.5	502.7
50	0.00084656	0.00011759	232.79	24.88	18829.7	506.8
60	0.00084312	0.00011856	235.15	30.66	18757.6	504.7
70	0.00084162	0.00008985	233.70	21.76	18745.5	504.2
80	0.00085887	0.00008526	228.99	17.61	18757.0	505.3
90	0.00085612	0.00007018	228.49	16.97	18826.8	507.4
100	0.00083800	0.00007228	233.07	16.13	18798.0	505.6

Summary Graph of % Mean B_0 Estimate differs from Γ_{a_0}

$\Gamma_{a_0} = 200$ $\Phi = 0.001$
 $n = 100$

Source : 200grp50
 Output : 200grp50.gph

%off



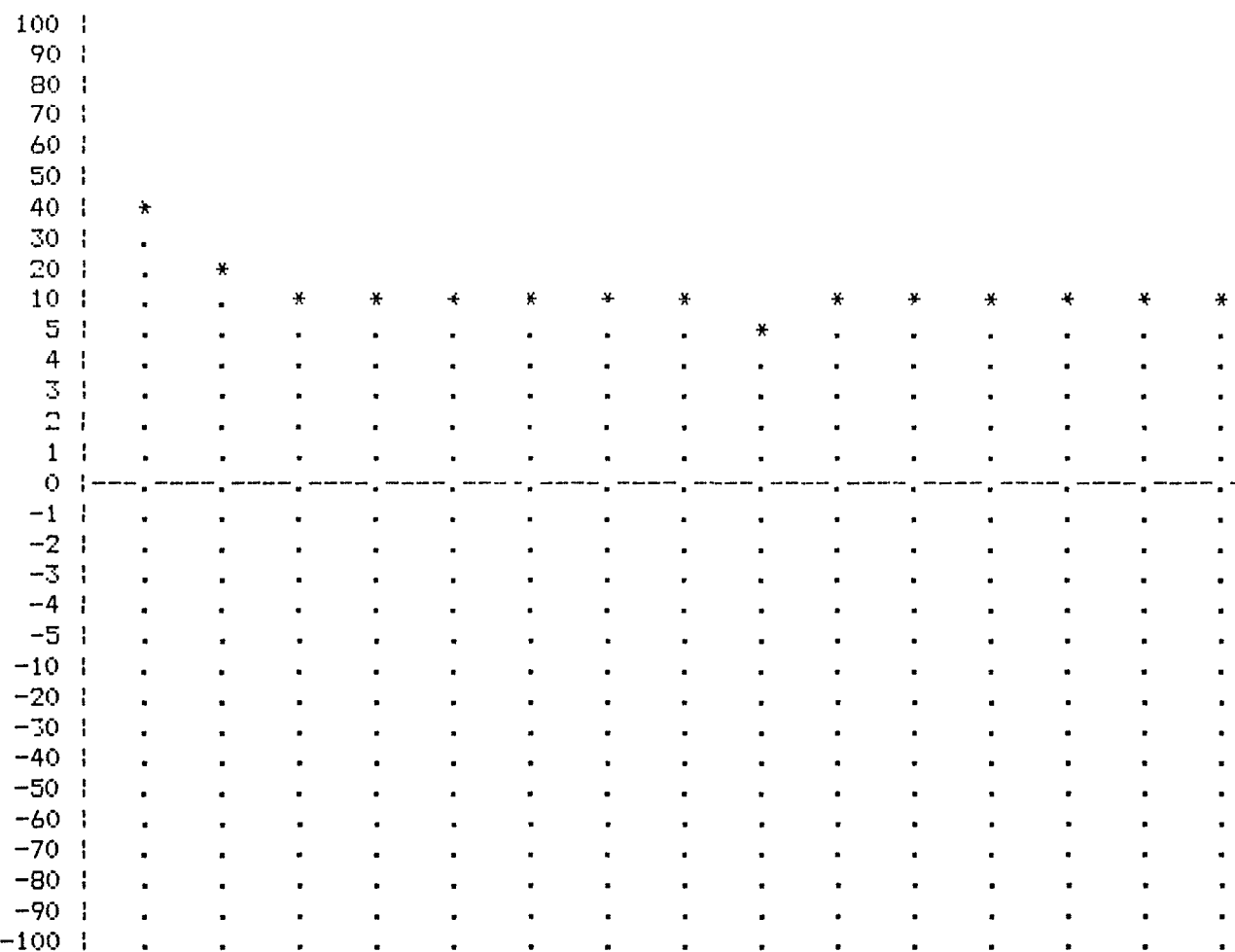
rep	b1		b0		tsum	te
	Mean	Std	Mean	Std	Mean	Mean
1	0.00091416	0.00054024	2645.41	9549.14	30494.9	681.8
2	0.00091737	0.00033314	249.56	109.74	30745.7	687.5
3	0.00094425	0.00025034	240.36	174.69	30561.0	684.5
4	0.00084019	0.00026279	245.76	65.01	31222.4	691.8
5	0.00088287	0.00023788	237.89	69.26	30784.9	684.8
10	0.00087394	0.00015452	227.20	29.78	30902.2	686.4
20	0.00090125	0.00012188	221.31	24.34	30608.8	681.7
30	0.00091719	0.00008909	216.31	16.76	30642.0	683.9
40	0.00091269	0.00008679	217.59	15.33	30541.3	680.9
50	0.00089587	0.00007175	219.01	13.19	30793.2	685.7
60	0.00090419	0.00006219	217.76	11.40	30675.3	683.5
70	0.00091100	0.00005478	216.32	9.82	30662.3	683.8
80	0.00091200	0.00006005	216.09	10.95	30689.4	684.6
90	0.00090375	0.00004145	216.65	7.27	30803.1	686.6
100	0.00089597	0.00005382	218.84	9.35	30728.7	684.0

Summary Graph of % Mean B_0 Estimate differs from Γ_0

$\Gamma_0 = 200$ $\Phi_1 = 0.001$
 $n = 120$

Source : 200grp60
 Output : 200grp60.gph

%off



1 2 3 4 5 1 2 3 4 5 6 7 8 9 1
 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 # replicates

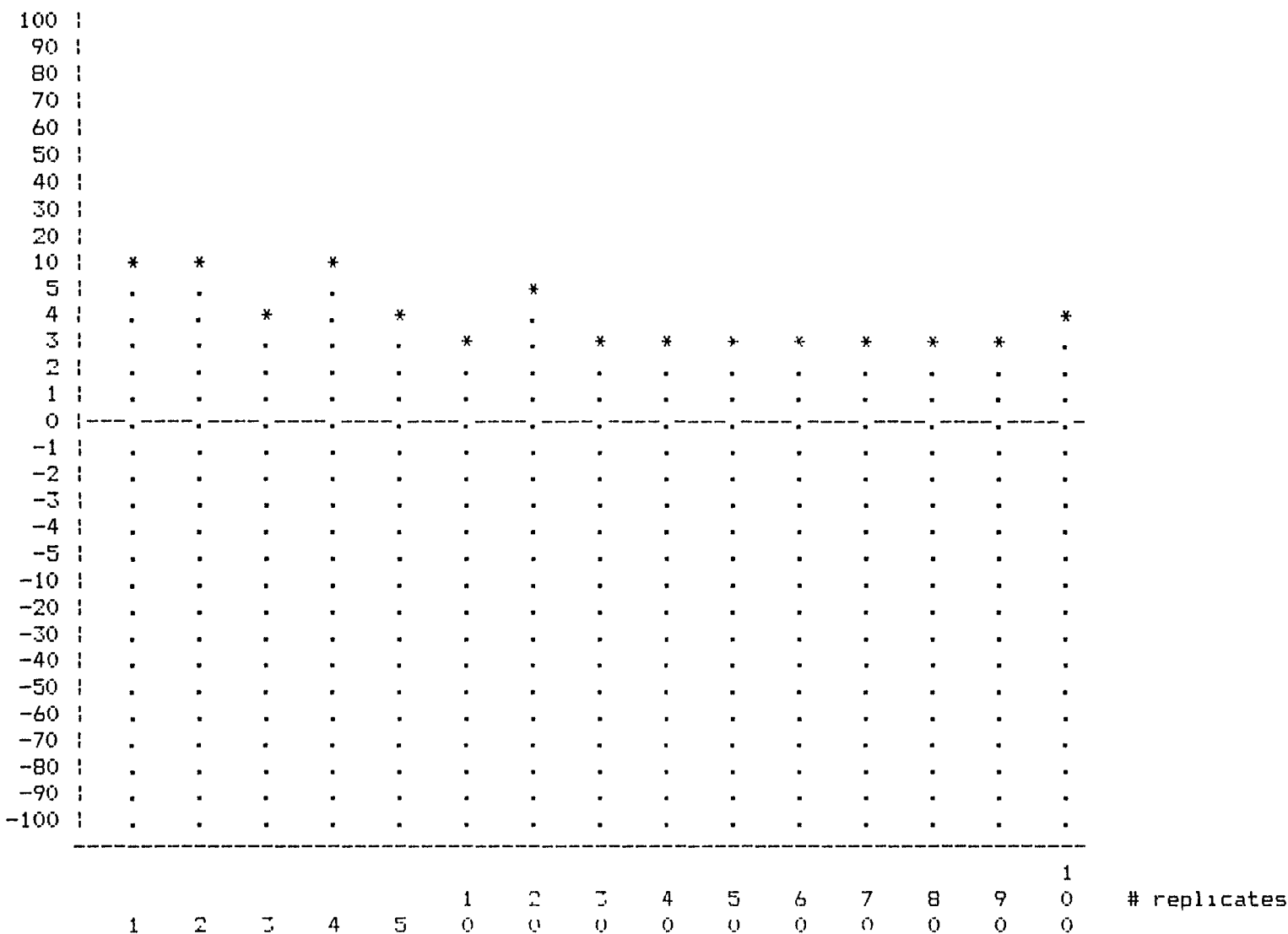
rep	b1		b0		tsum	te
	Mean	Std	Mean	Std	Mean	Mean
1	0.00094406	0.00040844	272.89	222.85	46400.6	903.9
2	0.00089298	0.00027900	243.88	89.54	46672.9	900.7
3	0.00094942	0.00021526	219.27	50.68	46492.0	904.0
4	0.00089628	0.00017962	222.39	36.59	47308.5	911.7
5	0.00092589	0.00016168	218.19	35.59	46657.0	903.0
10	0.00092447	0.00010592	213.80	17.18	46854.8	906.2
20	0.00092631	0.00008432	214.10	13.17	46412.9	896.2
30	0.00093897	0.00006673	211.06	10.22	46510.1	900.8
40	0.00094900	0.00006183	209.67	8.92	46378.8	899.4
50	0.00093739	0.00005378	210.24	7.66	46740.3	905.6
60	0.00094094	0.00004798	210.19	7.75	46559.6	902.2
70	0.00093728	0.00003811	210.74	6.16	46534.3	900.9
80	0.00093956	0.00004135	210.16	6.64	46601.4	902.9
90	0.00093641	0.00002975	210.01	4.55	46760.9	905.9
100	0.00093512	0.00003551	210.35	5.51	46422.5	902.5

Summary Graph of % Mean B_0 Estimate differs from Γ_0

$\Gamma_0 = 200$ $\Phi_1 = 0.001$
 $n = 140$

Source : 200grp70
 Output : 200grp70.gph

%off

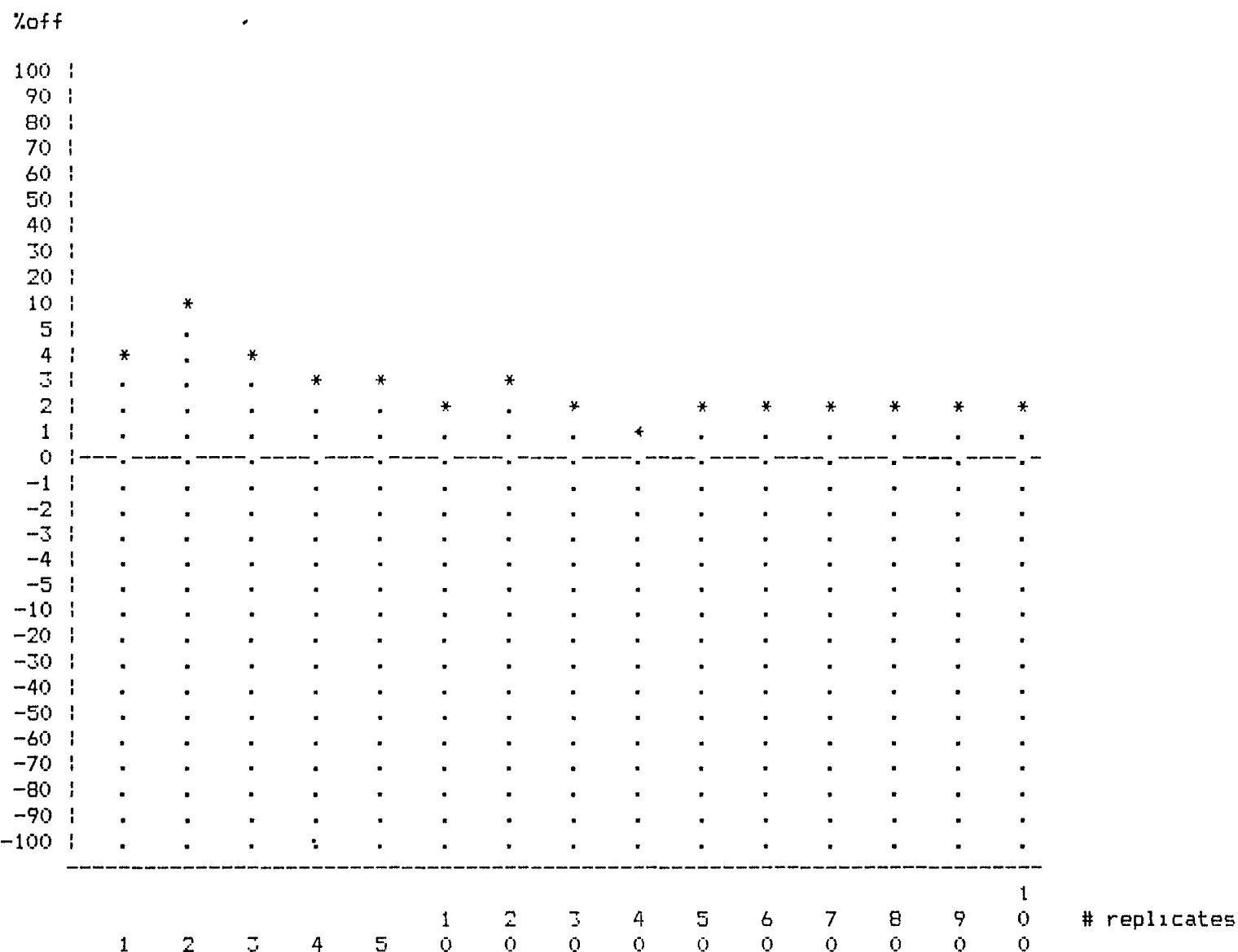


rep	b1		b0		tsum	te
	Mean	Std	Mean	Std	Mean	Mean
1	0.00094550	0.00024369	221.66	50.05	67287.9	1184.3
2	0.00091689	0.00018064	220.95	38.75	67485.5	1174.6
3	0.00098741	0.00017224	207.10	26.06	67490.3	1196.4
4	0.00090853	0.00011646	215.20	19.49	68333.5	1185.8
5	0.00096584	0.00011657	207.26	14.23	67583.7	1187.9
10	0.00096135	0.00007422	206.21	10.19	67864.7	1193.2
20	0.00095091	0.00005709	209.04	8.80	67095.3	1172.3
30	0.00096191	0.00004103	206.53	5.69	67333.1	1180.9
40	0.00096839	0.00004551	205.98	6.42	67170.9	1179.6
50	0.00095777	0.00003576	206.35	4.41	67668.3	1186.9
60	0.00095912	0.00003477	206.73	5.18	67398.2	1181.5
70	0.00095880	0.00003269	206.82	4.56	67363.3	1180.5
80	0.00095856	0.00002641	206.57	4.06	67474.1	1182.9
90	0.00095766	0.00002442	206.21	3.18	67695.0	1187.5
100	0.00095377	0.00002333	207.25	3.18	67471.8	1181.1

Summary Graph of % Mean B_0 Estimate differs from Γ_0

$\Gamma_0 = 200$ $\Phi_1 = 0.001$
 $n = 160$

Source : 200grp80
 Output : 200grp80.gph



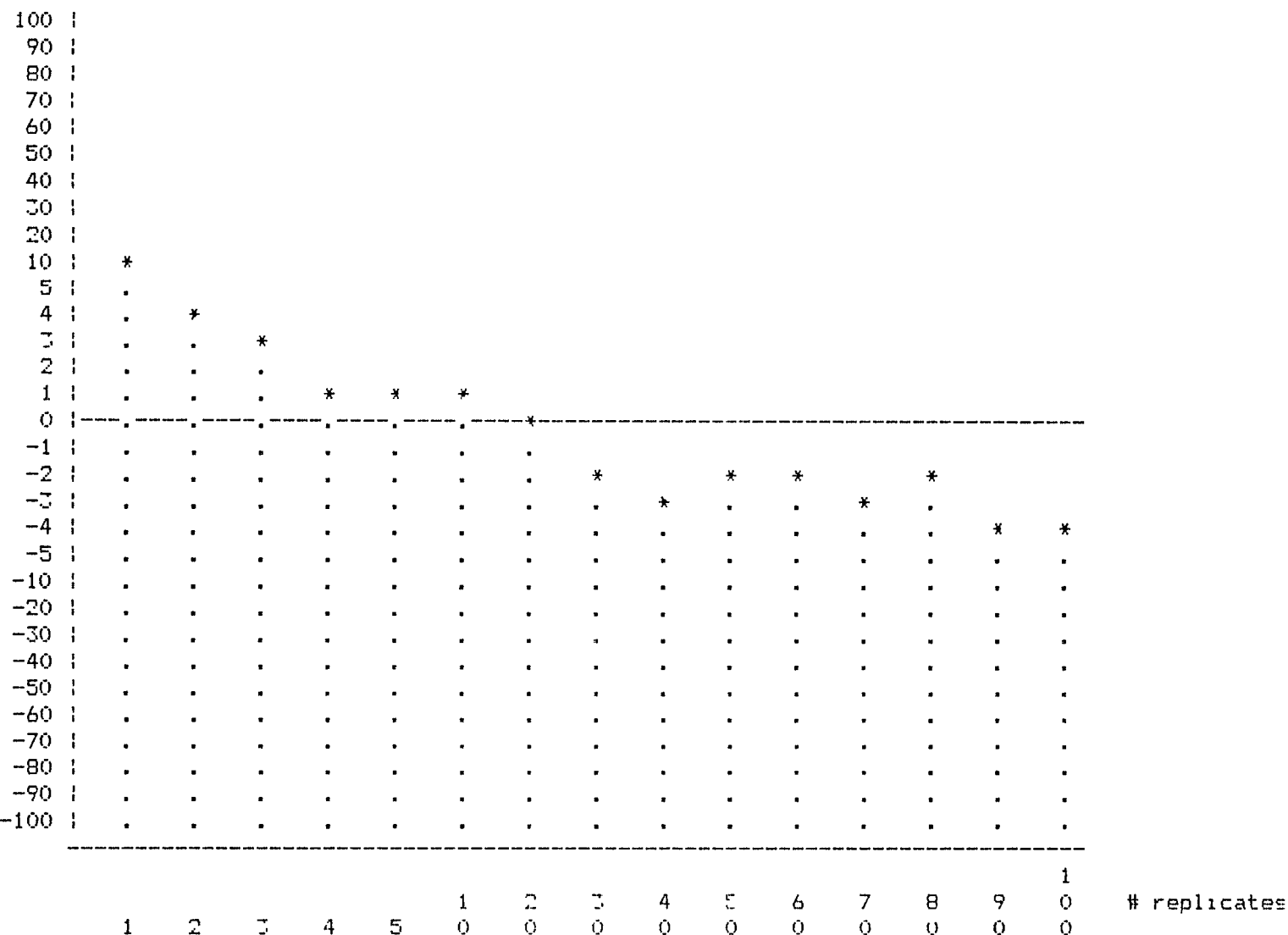
rep	b1		b0		tsum	te
	Mean	Std	Mean	Std	Mean	Mean
1	0.00097852	0.00015416	207.87	19.91	94830.3	1583.9
2	0.00094032	0.00012061	211.82	18.69	94779.0	1554.5
3	0.00096098	0.00011083	207.79	15.19	95144.6	1577.7
4	0.00095387	0.00008315	206.64	12.39	96085.3	1592.8
5	0.00097079	0.00007342	205.32	10.21	95144.4	1580.4
10	0.00096932	0.00005834	204.42	7.95	95653.8	1590.4
20	0.00096948	0.00004182	205.77	6.02	94356.1	1558.7
30	0.00097996	0.00003572	203.67	4.69	94830.9	1576.5
40	0.00098913	0.00003721	202.72	4.62	94684.5	1579.3
50	0.00097369	0.00002661	203.76	3.23	95295.4	1582.6
60	0.00097653	0.00002190	203.89	3.67	94904.5	1575.7
70	0.00097902	0.00002575	203.66	3.18	94836.3	1575.5
80	0.00097137	0.00002233	204.44	3.35	94978.6	1573.6
90	0.00097340	0.00001831	203.69	2.86	95340.0	1583.6
100	0.00097377	0.00001807	204.14	2.55	94947.9	1574.3

Summary Graph of % Mean B_0 Estimate differs from Γ_0

$\Gamma_0 = 200$ $\Phi_1 = 0.001$
 $n = 180$

Source : 200grp90
 Output : 200grp90.gph

%off



rep	b1		b0		tsum	te
	Mean	Std	Mean	Std	Mean	Mean
1	0.00094485	0.00012528	210.78	19.99	132685.0	2217.5
2	0.00096704	0.00010101	207.29	14.36	132312.5	2378.8
3	0.00095652	0.00007967	206.59	11.45	133918.4	2812.9
4	0.00097790	0.00006802	202.51	10.25	135761.3	2903.4
5	0.00099683	0.00006165	201.71	9.78	134113.1	2626.0
10	0.00098405	0.00006658	201.26	9.42	137118.5	3872.4
20	0.00101611	0.00009164	199.63	9.45	135684.3	4115.4
30	0.00104322	0.00008510	196.57	8.29	135774.6	3681.0
40	0.00107010	0.00009849	194.25	9.61	137002.2	4375.3
50	0.00106072	0.00009174	195.52	8.01	134940.5	2971.7
60	0.00105955	0.00008463	195.14	8.19	136009.0	3798.7
70	0.00107840	0.00008722	194.22	7.78	134664.9	3163.2
80	0.00106175	0.00008175	195.61	7.35	134094.5	2847.0
90	0.00110078	0.00010184	191.89	9.08	136978.8	3960.1
100	0.00110714	0.00010693	191.92	8.48	135786.1	3685.9

Appendix D

The figures contained in Appendix D, illustrate the gains from replication. For each of the total fault counts used (50, 100, 200), the number of replicates required for the B_0 estimate to be specific %s off from the actual parameter with the different number of failure times generated are charted.

Example. On page D-1, the entry '35' in the 50% column, 4 row, indicates that for 35 failure times simulated and 4 replicates used, the mean of the 50 estimates calculated is 50% off from the actual parameter value.

50 Total Program Faults: Grid entries are Number of Failures needed to be generated for the number of Replicates indicated to be %off indicated.

% Mean of 50 B₀ Estimates Differs from Actual Parameter

replicates	50%	40%	30%	20%	10%	5%	4%	3%	2%	1%	0%
1											
2			40	45							
3				40	45						
4	35										
5			35								
10		30		35	40						
20								45			
30			30								
40											
50									45		
60											
70											
80					35						45
90										45	
100	25									-45	

* '-' indicates that estimate is less than actual parameter, otherwise greater.

100 Total Program Faults: Grid entries are Number of Failures needed to be generated for the number of Replicates indicated to be %off indicated.

% Mean of 50 B₀ Estimates Differs from Actual Parameter

replicates	50%	40%	30%	20%	10%	5%	4%	3%	2%	1%	0%
1				70	80/90						
2			70								
3			60		70		90				
4											
5			50	60							
10					60	80					90
20		40		50							
30							80				
40	40								-90		
50											
60								-90			
70						-90		80			
80											
90											
100											

* '-' indicates that estimate is less than actual parameter, otherwise greater.

200 Total Program Faults: Grid entries are Number of Failures needed to be generated for the number of Replicates indicated to be %off indicated.

% Mean of 50 B₀ Estimates Differs from Actual Parameter

replicates	50%	40%	30%	20%	10%	5%	4%	3%	2%	1%	0%
1					140/180		160				
2				100/120	160		180				
3					120		140	180			
4								160		180	
5											
10			80		100			140	160		
20	60			80							180
30					80				-180		
40								-180			
50		60									
60											
70											
80											
90							-180				
100											

* '-' indicates that estimate is less than actual parameter, otherwise greater.

Appendix E

This Appendix contains graphs showing the results of Confidence Interval testing. 50, 55, 60, ..., 95% Confidence Intervals were calculated for 200 parameter estimates for each total fault count with 10, 20, ...90% of total program faults generated. Each graph column shows the percent of the 200 calculated confidence intervals which contained the actual parameter value. For example on page E1-1, the '#' in row 85 column 20 indicates that 85% of the the confidence intervals calculated using 20 simulated failure times contained the actual B_0 parameter value used to simulate the failure data. And, the @ in row 65 column 20 indicates that 65% of the confidence intervals calculated using 20 simulated failure times contained the actual B_1 parameter value used to simulate the failure data.

APPENDIX E INDEX

E1-1	Confidence Intervals for 50 total faults
E2-1	Confidence Intervals for 100 total faults
E3-1	Confidence Intervals for 200 total faults

Graph of % of E_0/B_1 Estimates Falling in 50% CI

= B_0 @ = B_1 * => $B_0 \geq B_1$

Gamma₀ = 50 Pn₁ = 0.0010 runs : 200

%good

100
95
90	*	*
85	.	.	*	#
80
75
70
65	.	.	.	@
60
55
50
45	@	#	.	.	.
40	#	@	*	@	@
35	#	.
30
25
20	#
15
10
5

5 1 1 2 2 3 3 4 4 # Failures Generated
0 5 0 5 0 5 0 5

Me	B_1	Mean B_1 Min	B_1 Max	%good	B_1 CI %high %low	B_0 CI %good	B_0	Mean B_0 min	B_0 max
005	0.001997	-0.011301	0.015295	0.940	0.060 0.000	0.935	1015.8	5.3	*
010	0.001151	-0.002756	0.005059	0.930	0.070 0.000	0.940	1008.7	11.6	*
015	0.000909	-0.001026	0.002843	0.885	0.105 0.010	0.895	1270.0	18.6	*
020	0.000817	-0.000342	0.001977	0.690	0.120 0.190	0.895	1223.9	26.1	*
025	0.000835	0.000074	0.001595	0.495	0.130 0.375	0.440	1171.8	33.7	*
030	0.000787	0.000235	0.001318	0.435	0.145 0.420	0.450	802.6	40.6	364.4
035	0.000793	0.000408	0.001177	0.420	0.130 0.450	0.440	463.6	46.4	127.0
040	0.000781	0.000496	0.001067	0.420	0.110 0.470	0.370	332.6	51.8	172.7
045	0.000809	0.000598	0.001019	0.440	0.055 0.505	0.205	237.4	54.8	86.5

* - indicates no upper bound for confidence interval.

Graph of % of B_0/B_1 Estimates Falling in 55% CI

=> B_0 @ => B_1 * => $B_0 \& B_1$

Gamma₀ = 50 Phi = 0.0010 runs : 200

%good

100
95	*
90	.	*	*	#
85
80	.	.	.	@
75
70
65
60
55	-----								
50	#	#	*	@	@
45	@	*	@	@
40	#	.
35
30
25
20	#
15
10
5

5 1 1 2 2 3 3 4 4 # Failures Generated
5 0 5 0 5 0 5 0 5

Me	Mean			%good	B_1 CI		B_0 CI	B_0	Mean	
	B_1	B_1 Min	B_1 Max		%high	%low			B_0 min	B_0 max
005	0.001997	-0.012877	0.016871	0.955	0.045	0.000	0.950	1015.8	5.3	*
010	0.001151	-0.003219	0.005322	0.940	0.060	0.000	0.940	1008.7	11.3	*
015	0.000907	-0.001255	0.003073	0.915	0.085	0.000	0.920	1270.0	18.2	*
020	0.000817	-0.000479	0.002114	0.830	0.090	0.080	0.915	1227.9	25.1	*
025	0.000835	-0.000016	0.001686	0.560	0.120	0.320	0.510	1171.6	32.5	*
030	0.000787	0.000192	0.001381	0.480	0.130	0.390	0.515	802.6	39.2	269.3
035	0.000795	0.000362	0.001225	0.480	0.100	0.420	0.470	463.6	45.6	6317.6
040	0.000781	0.000462	0.001101	0.450	0.100	0.450	0.415	332.6	50.5	303.9
045	0.000809	0.000573	0.001044	0.490	0.045	0.465	0.230	237.4	54.2	99.0

* - indicates no upper bound for confidence interval.

Graph of % of B_0/B_1 Estimates Falling in 60% CI

= B_0 @ = B_1 * = $B_0 \approx B_1$

Gamma₀ = 50 Phi = 0.0010 runs : 200

%good

100
95	*
90	.	*	*	#
85	.	.	.	@
80
75
70
65	@
60	-----								
55	*	.	.	.
50	#	.	*	.	@
45	*	.
40
35
30
25	#
20
15
10
5

5 1 1 2 2 3 3 4 4 # Failures Generated
0 0 0 0 0 0 0 0 0

Me	B_1	Mean B_1 Min	B_1 Max	%good	B_1 CI %high %low	B_1 CI %good	B_0	Mean B_0 min	B_0 max
005	0.001997	-0.014552	0.018546	0.960	0.040 0.000	0.960	1015.8	5.2	*
010	0.001151	-0.003711	0.006014	0.940	0.060 0.000	0.945	1008.7	10.9	*
015	0.000909	-0.001492	0.003316	0.925	0.075 0.000	0.925	1270.0	17.8	*
020	0.000817	-0.000625	0.002260	0.875	0.080 0.045	0.920	1223.9	24.5	*
025	0.000835	-0.000112	0.001781	0.655	0.100 0.245	0.535	1171.8	31.6	*
030	0.000767	0.000126	0.001448	0.550	0.115 0.335	0.575	802.6	37.8	116.1
035	0.000793	0.000314	0.001271	0.515	0.090 0.395	0.505	463.6	44.4	166.0
040	0.000781	0.000426	0.001137	0.485	0.095 0.420	0.460	332.6	49.2	112.2
045	0.000809	0.000547	0.001070	0.510	0.040 0.450	0.255	237.4	53.6	270.7

* - indicates no upper bound for confidence interval.

Graph of % of B_0/B_1 Estimates Falling in 65% CI

= B_0 @ = B_1 * = $B_0 \geq B_1$

Gamma₀ = 50 Phi = 0.0010 runs : 200

%good

100
95	*	*
90	.	.	*	*
85
80
75	@
70
65	-----								
50	*	.	.	.
55	#	.	@	.	.
60	#	*	.
45
40
35
30
25	#
20
15
10
5

5 0 5 0 5 0 5 4 5 # Failures Generated

Me	B_1	Mean B_1 min	B_1 Max	%good	B_1 CI %high	%low	B_1 CI %good	B_0	Mean B_0 min	B_0 max
005	0.001977	-0.016424	0.020417	0.960	0.040	0.000	0.960	1015.8	5.1	*
010	0.001151	-0.004261	0.006564	0.950	0.050	0.000	0.950	1008.7	10.9	*
015	0.000909	-0.001771	0.003588	0.945	0.055	0.000	0.940	1270.0	17.2	*
020	0.000817	-0.000788	0.002427	0.920	0.070	0.010	0.920	1225.9	23.7	*
025	0.000835	-0.000219	0.001888	0.755	0.100	0.145	0.565	1171.8	30.6	*
030	0.000787	0.000051	0.001523	0.615	0.095	0.290	0.505	802.6	37.0	129.9
035	0.000793	0.000260	0.001325	0.565	0.080	0.355	0.540	463.6	43.4	444.2
040	0.000781	0.000386	0.001177	0.530	0.085	0.385	0.515	332.6	48.5	173.4
045	0.000809	0.000517	0.001100	0.555	0.030	0.415	0.290	237.4	52.8	118.9

* - indicates no upper bound for confidence interval.

Graph of % of B_0/B_1 Estimates Falling in 70% CI

= B_0 @ = B_1 * = $B_0 \& B_1$

Gamma₀ = 50 Phi = 0.0010 runs : 200

%good

100
95	*	*	*
90	.	.	.	*	#
85
80	@
75
70	-----								
65	@	.	.	.
60	#	@	@	.
55	#	.	@
50	#	.
45
40
35
30	#
25
20
15
10
5

3 1 1 2 2 3 3 4 4 = Failures Generated
5 0 5 0 5 5 0 5

Me	B_1	Mean B_1 Min	B_1 Max	%good	B_1 CI %high %low	B_0 CI %good	B_0	Mean B_0 min	B_0 max
005	0.001997	-0.019394	0.022387	0.975	0.025 0.000	0.970	1013.8	5.1	*
010	0.001151	-0.004840	0.007143	0.970	0.030 0.000	0.970	1008.7	10.7	*
015	0.000909	-0.002057	0.003875	0.950	0.050 0.000	0.950	1270.0	16.8	*
020	0.000817	-0.000960	0.002595	0.925	0.065 0.010	0.935	1223.9	23.1	*
025	0.000835	-0.000331	0.002001	0.820	0.100 0.130	0.925	1171.8	29.8	*
030	0.000787	-0.000628	0.001601	0.685	0.075 0.140	0.675	802.6	36.3	*
035	0.000793	0.000203	0.001382	0.610	0.070 0.120	0.595	463.6	42.1	132.9
040	0.000781	0.000343	0.001220	0.605	0.055 0.140	0.545	332.6	47.4	143.1
045	0.000809	0.000486	0.001131	0.565	0.030 0.405	0.325	237.4	51.9	199.3

* - indicates no upper bound for confidence interval.

Graph of % of B_0/B_1 Estimates Falling in 75% CI

= B_0 @ = B_1 * => $B_0 > B_1$

Gamma₀ = 50 Phi = 0.0010 runs : 200

%good

100
95	*	*	*
90	.	.	.	*	#
85	@
80
75	-----@-----								
70
65	#	*	@	.
60	@
55	#	.
50
45
40
35	#
30
25
20
15
10
5

5 1 1 2 2 3 3 4 4 # Failures Generated
0 0 0 5 0 0 0 0 5

Me	Mean			%good	B_1 CI		B_0 CI	B_0	Mean	
	B_1	B_1 Min	B_1 Max		%high	%low			B_0 min	B_0 max
05	0.001997	-0.020659	0.024657	0.975	0.025	0.000	0.975	1015.8	5.1	*
10	0.001151	-0.005505	0.007808	0.975	0.025	0.000	0.975	1008.7	10.5	*
15	0.000909	-0.002387	0.004205	0.965	0.035	0.000	0.965	1270.0	16.4	*
20	0.000817	-0.001157	0.002792	0.945	0.050	0.005	0.940	1223.9	22.6	*
25	0.000835	-0.000461	0.002131	0.860	0.090	0.050	0.940	1171.8	28.7	*
30	0.000787	-0.000118	0.001692	0.770	0.045	0.185	0.670	902.6	35.4	*
35	0.000793	0.000137	0.001448	0.660	0.045	0.295	0.655	463.6	41.5	262.9
40	0.000781	0.000295	0.001268	0.650	0.035	0.315	0.585	332.6	46.5	169.7
45	0.000809	0.000450	0.001167	0.620	0.025	0.355	0.380	237.4	51.1	119.7

* - indicates no upper bound for confidence interval.

Graph of % of B_0/B_1 Estimates Falling in 80% CI

=> B_0 @ = B_1 * => B_0 & B_1

Gamma₀ = 50 Phi = 0.0010 runs : 200

%good

100
95	*	+	+	*	#
90	@
85
80	-----@-----								
75
70	#	+	.	.
65	@	@
60	#	.
55
50
45
40	#
35
30
25
20
15
10
5

5 1 1 2 2 3 3 4 4 # Failures Generated
5 0 5 0 5 0 5 0 5

Me	B_1	Mean E_1 Min	B_1 Max	%good	B_1 CI %high %low	B_0 CI %good	E_0	Mean B_0 min	B_0 max
005	0.001997	-0.023220	0.027214	0.995	0.005 0.000	0.985	1015.8	5.0	*
010	0.001151	-0.006258	0.008561	0.980	0.020 0.000	0.980	1008.7	10.3	*
015	0.000909	-0.002759	0.004577	0.975	0.025 0.000	0.970	1270.0	16.1	*
020	0.000817	-0.001381	0.003016	0.965	0.035 0.000	0.965	1223.9	22.0	*
025	0.000835	-0.000607	0.002277	0.930	0.060 0.010	0.950	1171.8	28.0	*
030	0.000787	-0.000221	0.001794	0.820	0.040 0.140	0.725	802.6	34.4	*
035	0.000793	0.000063	0.001522	0.715	0.030 0.255	0.700	463.6	40.5	199.4
040	0.000781	0.000240	0.001323	0.695	0.015 0.290	0.625	332.6	45.7	549.2
045	0.000809	0.000410	0.001207	0.660	0.010 0.330	0.435	237.4	50.4	150.4

* - indicates no upper bound for confidence interval.

Graph of % of B_0/B_1 Estimates Falling in 85% CI

=> B_0 @ => B_1 * => $B_0 \> B_1$

$\sigma_{B_0} = 50$ $\Phi_1 = 0.0010$ runs : 200

good

0
5	*	*	*	*	#
10	@	#	.	.	.
15	@	.	.	.
20	@	@	.
25	#	.	@
30	#	.
35
40
45	#
50
55
60
65
70
75
80
85
90
95
100

5 1 1 2 2 3 3 4 4 # Failures Generated
0 5 0 5 0 5 0 5

B_1	Mean B_1 Min	B_1 Max	%good	B_1 CI %high %low	B_0 CI %good	B_0	Mean B_0 min	B_0 max
0.001997	-0.026373	0.030366	0.995	0.005 0.000	0.995	1015.8	5.0	*
0.001151	-0.007184	0.009487	0.985	0.015 0.000	0.985	1008.7	10.2	*
0.000909	-0.003218	0.005036	0.975	0.025 0.000	0.990	1270.0	15.8	*
0.000817	-0.001655	0.003290	0.975	0.025 0.000	0.985	1223.9	21.4	*
0.000835	-0.000788	0.002457	0.940	0.050 0.010	0.970	1171.8	27.4	*
0.000787	-0.000347	0.001920	0.890	0.035 0.075	0.930	802.6	33.5	*
0.000793	-0.000028	0.001613	0.790	0.025 0.185	0.745	463.6	39.6	204.1
0.000781	0.000172	0.001391	0.750	0.005 0.245	0.665	332.6	44.8	170.5
0.000809	0.000360	0.001257	0.735	0.010 0.255	0.520	237.4	49.6	129.2

* - indicates no upper bound for confidence interval.

Graph of % of B_0/B_1 Estimates Falling in 90% CI

= B_0 @ = B_1 * = $B_0 \& B_1$

Gamma₀ = 50 Phi = 0.0010 runs : 200

%good

100	*	.	#
95	.	*	@	*	*	*	.	.	.
90	-----@-----								
85
80	#	@	.
75	@
70	#	.
65
60
55	#
50
45
40
35
30
25
20
15
10
5

Failures Generated

5 0 5 0 5 0 5 0 5

Me	B_1	Mean B_1 Min	B_1 Max	%good	B_1 CI %high %low	B_0 CI %good	B_0	Mean B_0 min	B_0 max
005	0.001997	-0.000411	0.0034405	1.000	0.000 0.000	1.000	1015.8	0.0	*
010	0.001151	-0.000371	0.010674	0.990	0.010 0.000	0.990	1008.7	10.1	*
015	0.000909	-0.0003805	0.005623	0.995	0.005 0.000	1.000	1270.0	15.6	*
020	0.000817	-0.000008	0.003643	0.995	0.005 0.000	0.990	1223.9	21.1	*
025	0.000835	-0.001019	0.002688	0.960	0.035 0.005	0.985	1171.8	26.8	*
030	0.000787	-0.000508	0.002081	0.970	0.015 0.015	0.990	802.6	32.9	*
035	0.000793	-0.000144	0.001730	0.900	0.010 0.090	0.815	463.6	38.6	*
040	0.000781	0.000085	0.001478	0.830	0.005 0.165	0.715	332.6	44.0	205.3
045	0.000809	0.000296	0.001321	0.780	0.010 0.210	0.555	237.4	48.8	182.4

* - indicates no upper bound for confidence interval.

Graph of % of B_0/B_1 Estimates Falling in 95% CI

= B_0 @ = B_1 * => $B_0 \approx B_1$

Gamma₀ = 50 Phi = 0.0010 runs : 200

%good

100	*	.	*	*	#
95					@	*	@		
90	@	.
85	#	.	@
80
75	#	.
70
65
60	#
55
50
45
40
35
30
25
20
15
10
5

5 1 1 2 2 3 3 4 4 # Failures Generated
5 0 5 0 5 0 5 0 5

Me	B_1	Mean B_1 Min	B_1 Max	%good	B_1 CI %high %low	B_0 CI %good	B_0	Mean B_0 min	B_0 max
005	0.001997	-0.036617	0.040611	1.000	0.000 0.000	1.000	1015.8	0.0	*
010	0.001151	-0.010195	0.012497	0.995	0.005 0.000	0.995	1008.7	10.0	*
015	0.000909	-0.004708	0.006526	1.000	0.000 0.000	1.000	1270.0	15.3	*
020	0.000817	-0.002549	0.004184	1.000	0.000 0.000	1.000	1223.9	20.6	*
025	0.000835	-0.001774	0.003043	0.990	0.010 0.000	1.000	1171.8	26.1	*
030	0.000787	-0.000756	0.002329	0.985	0.005 0.010	0.995	802.6	31.9	*
035	0.000793	-0.000324	0.001909	0.980	0.000 0.020	0.885	463.6	37.4	*
040	0.000781	-0.000048	0.001611	0.935	0.000 0.065	0.790	332.6	43.0	300.4
045	0.000809	0.000198	0.001419	0.880	0.010 0.110	0.645	237.4	47.8	174.8

* - indicates no upper bound for confidence interval.

Graph of % of E_0/B_1 Estimates Falling in 50% CI

= E_0 @ = B_1 * = B_0 & B_1

Gamma₀ = 100 Phi = 0.0010 runs : 200

%good

100
95
90
85	*	*	#
80	.	.	@
75
70
65
60
55
50	-----	-----	-----	-----	-----	-----	-----	-----	-----
45	.	.	.	#	.	*	.	.	.
40	#	.	#	.	@
35
30	#	.
25
20	#
15
10
5

1 2 3 4 5 6 7 8 9 # Failures Generated
0 1 0 0 0 0 0 0 0

Me	E_1	Mean B_1 Min	B_1 Max	%good	B_1 CI		E_0 CI	B_0	Mean B_0 min	B_0 max
					%high	%low	%good			
010	0.002269	-0.005804	0.010343	0.885	0.115	0.000	0.885	1650.4	11.7	*
020	0.001337	-0.001257	0.003971	0.850	0.150	0.000	0.855	2011.7	26.5	*
030	0.000981	-0.000335	0.002298	0.810	0.155	0.035	0.850	2445.1	43.1	*
040	0.000942	0.000157	0.001727	0.525	0.175	0.300	0.475	1722.4	59.8	324.5
050	0.000841	0.000317	0.001366	0.505	0.140	0.355	0.440	1384.3	76.8	433.6
060	0.000905	0.000542	0.001269	0.480	0.175	0.345	0.480	606.3	89.9	480.5
070	0.000914	0.000653	0.001176	0.505	0.155	0.340	0.425	204.3	99.5	214.7
080	0.000919	0.000727	0.001109	0.510	0.130	0.360	0.330	122.9	103.1	142.3
090	0.000920	0.000782	0.001057	0.435	0.135	0.430	0.210	111.2	105.0	121.7

* - indicates no upper bound for confidence interval.

Graph of % of B_0/B_1 Estimates Falling in 53% CI

=> B_0 @ => B_1 * => $B_0 \geq B_1$

Gamma₀ = 100 Phi = 0.0010 runs : 200

%good

100
95
90	*
85	.	*	*
80
75
70
65
60	.	.	.	@
55	-----@-----@-----@-----								
50	.	.	.	#	#	*	.	.	.
45	#	.	@
40
35	#	.
30
25
20	#
15
10
5

	1	2	3	4	5	6	7	8	9
	0	0	0	0	0	0	0	0	0

Failures Generated

Me	Mean			%good	B_1 CI		B_0 CI	B_0	Mean	
	B_1	B_1 Min	B_1 Max		%high	%low			B_0 min	B_0 max
010	0.002267	-0.006761	0.011300	0.905	0.095	0.000	0.900	1650.4	11.4	*
020	0.001357	-0.001567	0.004250	0.875	0.125	0.000	0.880	2011.7	25.3	*
030	0.000981	-0.000491	0.002454	0.855	0.130	0.015	0.870	2445.1	41.4	*
040	0.000742	0.000064	0.001821	0.805	0.165	0.230	0.535	1722.4	57.6	*
050	0.000841	0.000254	0.001428	0.585	0.105	0.310	0.510	1384.3	73.6	459.9
060	0.000906	0.000499	0.001312	0.545	0.155	0.300	0.530	606.3	87.7	263.7
070	0.000914	0.000622	0.001207	0.550	0.145	0.305	0.475	204.3	97.4	235.7
080	0.000918	0.000704	0.001131	0.530	0.110	0.340	0.350	122.9	102.1	149.8
090	0.000920	0.000766	0.001073	0.475	0.120	0.405	0.240	111.2	104.5	123.6

* - indicates no upper bound for confidence interval.

Graph of % of B_0/B_1 Estimates Falling in 60% CI

=> B_0 @ => B_1 * => $B_0 \& B_1$

Gamma₀ = 100 Fh₁ = 0.0010 runs : 200

%good

100
95
90	*
85	.	*	*
80
75
70	.	.	.	@
65
60	@	@	.	@	.
55	.	.	.	#	#	#	@	.	.
50	#	.	@
45
40	#	.
35
30
25	#
20
15
10
5

1 2 3 4 5 6 7 8 9 # Failures Generated
0 0 0 0 0 0 0 0 0

Va	B_1	Mean		%good	B_1 CI		B_0 CI	B_0	Mean	
		B_1 Min	B_1 Max		%high	%low			B_0 min	B_0 max
010	0.002269	-0.007777	0.012316	0.920	0.080	0.000	0.925	1650.4	11.2	*
020	0.001357	-0.001896	0.004610	0.895	0.105	0.000	0.885	2011.7	24.6	*
030	0.000981	-0.000657	0.002620	0.880	0.120	0.000	0.885	2445.1	39.8	*
040	0.000942	-0.000035	0.001919	0.740	0.125	0.135	0.590	1722.4	55.8	*
050	0.000841	0.000189	0.001494	0.635	0.100	0.265	0.560	1384.3	71.5	294.6
060	0.000906	0.000453	0.001358	0.600	0.130	0.270	0.590	606.3	85.6	983.3
070	0.000914	0.000589	0.001240	0.590	0.120	0.290	0.510	204.3	95.5	563.8
080	0.000918	0.000680	0.001155	0.605	0.090	0.305	0.410	122.9	101.1	164.7
090	0.000920	0.000748	0.001091	0.535	0.095	0.370	0.255	111.2	103.9	125.8

* - indicates no upper bound for confidence interval.

Graph of % of B_0/B_1 Estimates Falling in 65% CI

= B_0 @ => B_1 * => B_0 & B_1

Gamma₀ = 100 Phi = 0.0010 runs : 200

%good

100
95
90	*	*	#
85	.	.	@	#
80	.	.	.	@
75
70
65	-----@-----@-----								
60	#	#	@	@	.
55	@
50	#	.	.
45
40	#	.
35
30
25	#
20
15
10
5

1 2 3 4 5 6 7 8 9 # Failures Generated
0 0 0 0 0 0 0 0 0

Me	E_1	Mean E_1 Min	B_1 Max	%good	B_1 CI %high %low	B_0 CI %good	B_0	Mean B_0 min	B_0 max
010	0.002269	-0.008914	0.013452	0.935	0.065 0.000	0.930	1650.4	10.9	*
020	0.001357	-0.002264	0.004977	0.915	0.085 0.000	0.905	2011.7	23.9	*
030	0.000981	-0.000842	0.002805	0.890	0.110 0.000	0.900	2445.1	38.0	*
040	0.000942	-0.000146	0.002030	0.845	0.105 0.050	0.890	1722.4	54.2	*
050	0.000841	0.000114	0.001568	0.655	0.100 0.245	0.630	1384.7	69.5	293.5
060	0.000502	0.000402	0.001409	0.655	0.120 0.225	0.625	606.3	83.1	800.9
070	0.000914	0.000552	0.001277	0.625	0.100 0.275	0.540	204.7	93.6	267.8
080	0.000918	0.000653	0.001182	0.625	0.080 0.295	0.430	122.9	99.7	165.4
090	0.000920	0.000729	0.001110	0.585	0.085 0.330	0.265	111.2	103.3	128.9

* - indicates no upper bound for confidence interval.

Graph of % of B_0/B_1 Estimates Falling in 70% CI

= B_0 @ = B_1 * = $B_0 \& B_1$

$\Gamma_{00} = 100$ $F_{01} = 0.0010$ runs : 200

%good

100
95	*
90	.	*	*	*
85
80
75
70	-----@-----								
65	*	*	@	.	.
60	@
55	#	.	.
50
45	#	.
40
35
30
25	#
20
15
10
5

	1	2	3	4	5	6	7	8	9
	0	0	0	0	0	0	0	0	0

Failures Generated

Me	B_1	Mean		%good	B_1 CI		E_0 CI	B_0	Mean	
		E_1 Min	B_1 Max		%high	%low			E_0 min	B_0 max
010	0.002269	-0.010110	0.014649	0.950	0.050	0.000	0.955	1650.4	10.8	*
020	0.001357	-0.002652	0.005365	0.920	0.080	0.000	0.915	2011.7	23.1	*
030	0.000981	-0.001037	0.003000	0.910	0.090	0.000	0.920	2445.1	36.8	*
040	0.000942	-0.000262	0.002146	0.900	0.085	0.015	0.915	1722.4	52.7	*
050	0.000841	0.000037	0.001646	0.695	0.085	0.220	0.680	1384.3	67.6	513.0
060	0.000906	0.000348	0.001463	0.695	0.105	0.200	0.675	606.3	81.2	636.8
070	0.000914	0.000513	0.001315	0.680	0.090	0.230	0.575	204.3	92.0	334.6
080	0.000918	0.000625	0.001210	0.705	0.070	0.225	0.490	122.9	98.7	557.8
090	0.000920	0.000709	0.001130	0.630	0.075	0.295	0.295	111.2	102.6	133.5

* - indicates no upper bound for confidence interval.

Graph of % of B_0/B_1 Estimates Falling in 75% CI

= B_0 @ = B_1 * = $B_0 \approx B_1$

Gamma₀ = 100 Phi = 0.0010 runs : 200

%good

100
95	*
90	.	*	*	*
85
80
75	-----*								
70	#	@	@	.
65	@
60	#	.	.
55	#	.
50
45
40
35
30	#
25
20
15
10
5

1 2 3 4 5 6 7 8 9 # Failures Generated
0 0 0 0 0 0 0 0 0

Me	B_1	Mean B_1 Min	B_1 Max	%good	B_1 CI %high %low	B_0 CI %good	B_0	Mean B_0 min	B_0 max
010	0.002269	-0.011485	0.016024	0.960	0.040 0.000	0.955	1650.4	10.6	*
020	0.001357	-0.003097	0.005810	0.930	0.070 0.000	0.940	2011.7	22.7	*
030	0.000981	-0.001262	0.003225	0.920	0.080 0.000	0.940	2445.1	36.0	*
040	0.000942	-0.000396	0.002290	0.915	0.075 0.010	0.935	1722.4	50.6	*
050	0.000841	-0.000053	0.001735	0.780	0.070 0.150	0.750	1384.3	65.5	*
060	0.000906	0.000286	0.001525	0.760	0.095 0.145	0.725	606.3	78.8	370.9
070	0.000914	0.000469	0.001360	0.730	0.080 0.190	0.645	204.3	89.8	258.3
080	0.000918	0.000593	0.001243	0.745	0.070 0.185	0.550	122.9	97.2	177.9
090	0.000920	0.000685	0.001154	0.695	0.055 0.250	0.335	111.2	102.0	146.2

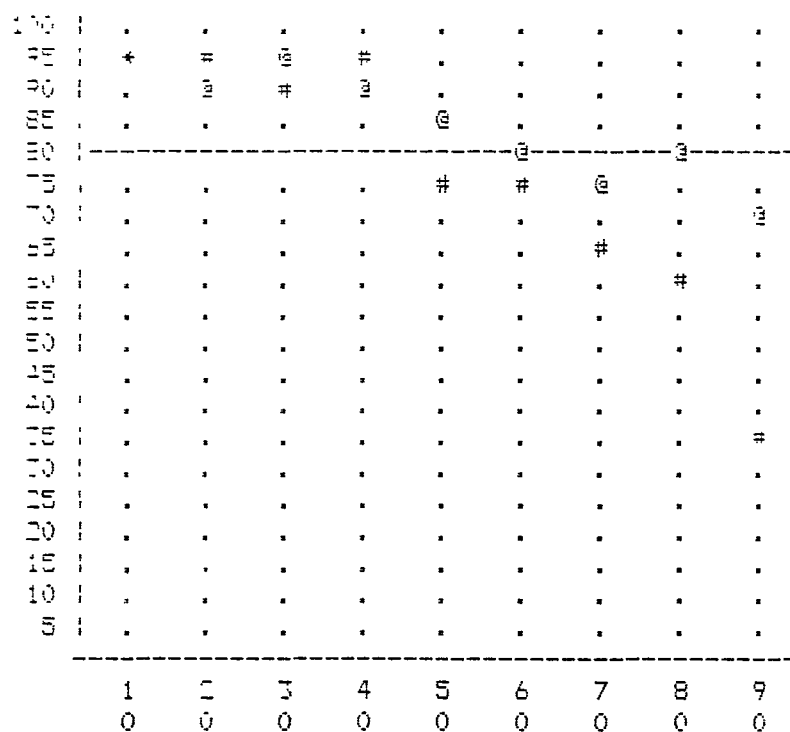
* - indicates no upper bound for confidence interval.

Graph of % of B_0/B_1 Estimates Falling in 80% CI

= B_0 @ = B_1 * = $B_0 \& B_1$

$\text{Gamma}_0 = 100$ $\text{Phi} = 0.0010$ runs : 200

%good



Me	Mean			%good	B_1 CI		B_0 CI	B_0	Mean	
	B_1	B_1 Min	B_1 Max		%high	%low			E_0 min	E_0 max
010	0.001269	-0.013040	0.017579	0.985	0.015	0.000	0.985	1650.4	10.5	*
020	0.001357	-0.003600	0.006314	0.945	0.053	0.000	0.955	2011.7	22.1	*
030	0.000981	-0.001515	0.003478	0.950	0.050	0.000	0.945	2445.1	34.8	*
040	0.000942	-0.000547	0.002431	0.940	0.060	0.000	0.955	1722.4	46.9	*
050	0.000841	-0.000154	0.001836	0.895	0.035	0.070	0.775	1384.3	65.7	*
060	0.000906	0.000216	0.001595	0.800	0.075	0.125	0.765	606.3	76.4	385.1
070	0.000914	0.000418	0.001410	0.795	0.065	0.140	0.585	204.3	88.2	499.7
080	0.000919	0.000536	0.001280	0.810	0.035	0.155	0.620	122.9	96.1	258.8
090	0.000920	0.000659	0.001180	0.735	0.040	0.225	0.370	111.2	101.0	134.5

* - indicates no upper bound for confidence interval.

Graph of % of B_0/B_1 Estimates Falling in 85% CI

= B_0 @ = B_1 * = $B_0 \& B_1$

Gamma₀ = 100 Phi = 0.0010 runs : 200

%good

100	
95	*	*	*	*	#	
90	@	
85						@				
80	#	@	@	.	
75	#	.	@	
70	
65	#	.	
60	
55	
50	
45	
40	#	
35	
30	
25	
20	
15	
10	
5	
	1	2	3	4	5	6	7	8	9	# Failures Generated
	0	0	0	0	0	0	0	0	0	

Graph of % of B_0/B_1 Estimates Falling in 90% CI

= B_0 @ = B_1 + => $B_0 \geq B_1$

Gamma₀ = 100 P=1 = 0.0010 runs : 200

%good

100	+
75	.	+	+	+	+
90	-----@-----								
35	#	@	@	@
30	#	.	.
75	#	.
70
65
60
55
50
45	#
40
35
30
25
20
15
10
5

	1	2	3	4	5	6	7	8	9
	0	0	0	0	0	0	0	0	0
	# Failures Generated								

Me	B_1	Mean		%good	B_1 CI		B_0 CI	B_0	Mean	
		B_1 Min	B_1 Max		%high	%low			B_0 min	B_0 max
010	0.002269	-0.017406	0.021944	1.000	0.000	0.000	1.000	1650.4	0.0	*
020	0.001357	-0.005014	0.007727	0.985	0.015	0.000	0.985	2011.7	21.1	*
030	0.000981	-0.002227	0.004190	0.975	0.025	0.000	0.975	2445.1	32.9	*
040	0.000942	-0.000972	0.002856	0.975	0.025	0.000	0.980	1722.4	45.9	*
050	0.000841	-0.000438	0.002120	0.970	0.025	0.005	0.970	1384.3	58.6	*
060	0.001906	0.000019	0.001792	0.910	0.030	0.060	0.870	606.3	71.4	812.7
070	0.000914	0.000277	0.001552	0.875	0.030	0.095	0.840	204.3	82.6	320.2
080	0.000918	0.000453	0.001383	0.895	0.025	0.030	0.760	122.9	92.6	209.0
090	0.000920	0.000585	0.001255	0.860	0.020	0.120	0.485	111.2	99.4	151.1

* - indicates no lower bound for confidence interval.

Graph of % of B_0/B_1 Estimates Falling in 95% CI

= B_0 @ = B_1 * = $B_0 \& B_1$

Gamma₀ = 100 Phi = 0.0010 runs : 200

%good

100	*	*
95			*	*	*	@	@	@	@
90	#	#	.	@
85	#	.
80
75
70
65
60
55	#
50
45
40
35
30
25
20
15
10
5

1 2 3 4 5 6 7 8 9 # Failures Generated
0 0 0 0 0 0 0 0 0

Me	B_1	Mean B_1 Min	B_1 Max	%good	B_1 CI %high %low	B_0 CI %good	B_0	Mean B_0 min	B_0 max
010	0.002269	-0.021173	0.025712	1.000	0.000 0.000	1.000	1650.4	0.0	*
020	0.001337	-0.006234	0.008947	1.000	0.000 0.000	1.000	2011.7	20.7	*
030	0.000981	-0.002842	0.004805	0.995	0.005 0.000	0.985	2445.1	31.8	*
040	0.000942	-0.001339	0.003223	0.995	0.005 0.000	0.995	1722.4	43.8	*
050	0.000841	-0.000682	0.002365	0.985	0.015 0.000	0.985	1384.3	55.9	*
060	0.000906	-0.000150	0.001961	0.970	0.020 0.010	0.920	606.3	68.6	*
070	0.000914	0.000155	0.001674	0.950	0.010 0.040	0.900	204.3	79.8	348.8
080	0.000918	0.000364	0.001472	0.950	0.005 0.045	0.860	122.9	90.6	911.8
090	0.000920	0.000520	0.001319	0.930	0.000 0.070	0.580	111.2	98.0	171.2

* - indicates no upper bound for confidence interval.

Graph of % of B_0/B_1 Estimates Falling in 50% CI

= B_0 @ = B_1 * = $B_0 \& B_1$

Gamma₀ = 200 Phi = 0.0010 runs : 200

%good

100
95
90
85	*	*
80
75
70
65
60
55
50	-----@-----+-----@-----								
45	.	.	=	*	@	#	.	@	@
40	@	#	.
35	#	.	.
30
25	#
20
15
10
5

				1	1	1	1	1	
2	4	6	8	0	2	4	6	8	# Failures Generated
0	0	0	0	0	0	0	0	0	

Me	B_1	Mean		%good	B_1 CI		B_0 CI	B_0	Mean	
		B_1 Min	B_1 Max		%high	%low			B_0 min	B_0 max
020	0.001881	-0.003354	0.007116	0.885	0.115	0.000	0.890	2886.5	27.4	*
040	0.000989	-0.000749	0.002727	0.875	0.125	0.000	0.870	4825.7	63.1	*
060	0.000742	0.000059	0.001825	0.540	0.195	0.265	0.450	4562.7	100.0	*
080	0.000863	0.000328	0.001397	0.465	0.185	0.350	0.460	2669.0	142.1	777.3
100	0.000915	0.000562	0.001268	0.485	0.180	0.335	0.505	941.9	175.0	572.2
120	0.000938	0.000692	0.001184	0.505	0.170	0.325	0.465	271.7	193.6	362.8
140	0.000965	0.000789	0.001140	0.420	0.220	0.360	0.355	222.0	198.4	269.6
160	0.000949	0.000821	0.001076	0.455	0.175	0.370	0.400	215.7	202.5	235.7
180	0.000936	0.000844	0.001027	0.460	0.125	0.415	0.270	212.7	206.0	221.8

* - indicates no upper bound for confidence interval.

Graph of % of B_0/B_1 Estimates Falling in 55% CI

= B_0 @ = B_1 * = $B_0 \& B_1$

Gamma₀ = 200 Phi = 0.0010 runs : 200

%good

100
95
90	.	@
85	*	#
80
75
70
65
60
55	-----	-----	-----	-----	-----	-----	-----	-----	-----
50	.	.	=	*	@	*	.	@	@
45	@	.	.
40	#	#	.
35
30	#
25
20
15
10
5

2	4	6	8	1	1	1	1	1	
0	0	0	0	0	0	0	0	0	# Failures Generated

Me	B_1	Mean B_1 Min	B_1 Max	%good	B_1 CI %high %low	B_0 CI %good	B_0	Mean B_0 min	B_0 max
020	0.001681	-0.003974	0.007736	0.895	0.105 0.000	0.895	2886.5	26.2	*
040	0.000939	-0.000955	0.002934	0.900	0.100 0.000	0.890	4825.7	59.3	*
060	0.000942	-0.000045	0.001929	0.695	0.170 0.135	0.505	4562.7	96.0	*
080	0.000863	0.000265	0.001461	0.520	0.155 0.325	0.505	2669.0	136.2	845.1
100	0.000915	0.000520	0.001310	0.510	0.180 0.310	0.550	941.9	169.4	921.0
120	0.000938	0.000663	0.001213	0.540	0.160 0.300	0.520	271.7	190.3	427.5
140	0.000965	0.000768	0.001161	0.450	0.215 0.335	0.420	222.0	196.3	281.3
160	0.000949	0.000806	0.001092	0.505	0.155 0.340	0.420	215.7	201.2	238.8
180	0.000936	0.000833	0.001038	0.515	0.110 0.375	0.300	212.7	205.3	223.0

* - indicates no upper bound for confidence interval.

Graph of % of B_0/B_1 Estimates Falling in 50% CI

$$\# = B_0 \quad @ = B_1 \quad * = B_0 \& B_1$$

Gamma₀ = 200 Phi = 0.0010 runs : 200

%good

100
95
90	+	+
85	.	.	#
80
75	.	.	@
70
65
60
55	.	.	.	*	.	*	.	@	@
50	@	.	.
45	#	#	.
40
35
30	#
25
20
15
10
5

2	4	6	8	1	1	1	1	1	
0	0	0	0	0	0	0	0	0	# Failures Generated

me	B ₁	Mean B ₁ Min	B ₁ Max	%good	B ₁ CI %high	%low	B ₀ CI %good	B ₀	Mean B ₀ min	B ₀ max
020	0.001881	-0.004634	0.008396	0.910	0.090	0.000	0.915	2886.5	25.0	*
040	0.000989	-0.001174	0.003152	0.905	0.095	0.000	0.915	4825.7	56.6	*
060	0.000942	-0.000157	0.002041	0.785	0.140	0.075	0.865	4562.7	93.3	*
080	0.000863	0.000197	0.001528	0.565	0.130	0.305	0.550	2669.0	130.3	1108.8
100	0.000915	0.000475	0.001355	0.610	0.155	0.235	0.605	941.9	163.0	752.1
120	0.000933	0.000632	0.001244	0.590	0.155	0.255	0.565	271.7	186.5	723.4
140	0.000965	0.000746	0.001183	0.500	0.190	0.310	0.480	222.0	194.1	302.3
160	0.000749	0.000790	0.001108	0.550	0.145	0.305	0.455	215.7	199.9	242.4
180	0.000936	0.000822	0.001050	0.565	0.090	0.345	0.330	212.7	204.6	224.5

* - indicates no upper bound for confidence interval.

Graph of % of B_0/B_1 Estimates Falling in 65% CI

= B_0 @ = B_1 * = $B_0 \geq B_1$

Gamma₀ = 200 Phi = 0.0010 runs : 200

%good

100
95
90	*	*
85	.	.	*
80
75
70
65	-----*								
60	.	.	.	*	.	@	.	@	@
55	#	@	.	.
50	#	.	.
45	#	.
40
35	#
30
25
20
15
10
5

2	4	6	8	1	1	1	1	1	
0	0	0	0	0	0	0	0	0	# Failures Generated

Me	B_1	Mean B_1 Min	B_1 Max	%good	B_1 CI %high %low	B_0 CI %good	B_0	Mean B_0 min	B_0 max
020	0.001881	-0.005370	0.009132	0.935	0.065 0.000	0.935	2886.5	24.4	*
040	0.000989	-0.001419	0.003397	0.920	0.080 0.000	0.920	4825.7	54.2	*
060	0.000942	-0.000281	0.002165	0.860	0.110 0.030	0.885	4562.7	88.8	*
080	0.000863	0.000122	0.001603	0.615	0.100 0.285	0.635	2669.0	124.2	642.0
100	0.000915	0.000426	0.001404	0.665	0.135 0.200	0.660	941.9	158.2	710.6
120	0.000938	0.000597	0.001279	0.620	0.140 0.240	0.585	271.7	182.3	569.9
140	0.000965	0.000721	0.001208	0.560	0.180 0.260	0.510	222.0	191.9	9969.3
160	0.000949	0.000772	0.001126	0.605	0.130 0.265	0.475	215.7	198.5	246.8
180	0.000936	0.000809	0.001062	0.635	0.070 0.295	0.355	212.7	203.8	226.1

* - indicates no upper bound for confidence interval.

Graph of % of B_0/B_1 Estimates Falling in 70% CI

= B_0 @ => B_1 * => $B_0 \notin B_1$

Gamma₀ = 200 Phi = 0.0010 runs : 200

%good

100
95	*
90	.	*	*
85
80
75
70	*
65	.	.	.	*	.	@	.	@	@
60	#	@	.	.
55	#	.	.
50	#	.
45
40
35	#
30
25
20
15
10
5

				1	1	1	1	1	
2	4	6	8	0	2	4	6	8	# Failures Generated
0	0	0	0	0	0	0	0	0	

Me	B_1	Mean B_1 Min	B_1 Max	%good	B_1 CI %high	%low	B_0 CI %good	B_0	Mean B_0 min	B_0 max
020	0.001881	-0.006146	0.009908	0.950	0.050	0.000	0.950	2886.5	23.7	*
040	0.000989	-0.001676	0.003655	0.935	0.065	0.000	0.930	4825.7	52.6	*
060	0.000942	-0.000412	0.002296	0.900	0.100	0.000	0.905	4562.7	85.7	*
080	0.000863	0.000043	0.001682	0.685	0.090	0.225	0.680	2669.0	120.5	*
100	0.000915	0.000373	0.001457	0.725	0.100	0.175	0.720	941.9	153.0	746.2
120	0.000938	0.000561	0.001315	0.665	0.115	0.220	0.645	271.7	177.5	728.4
140	0.000965	0.000695	0.001234	0.605	0.165	0.230	0.585	222.0	189.0	308.8
160	0.000949	0.000753	0.001145	0.655	0.110	0.235	0.500	215.7	197.1	251.9
180	0.000936	0.000795	0.001076	0.665	0.065	0.270	0.370	212.7	203.0	228.0

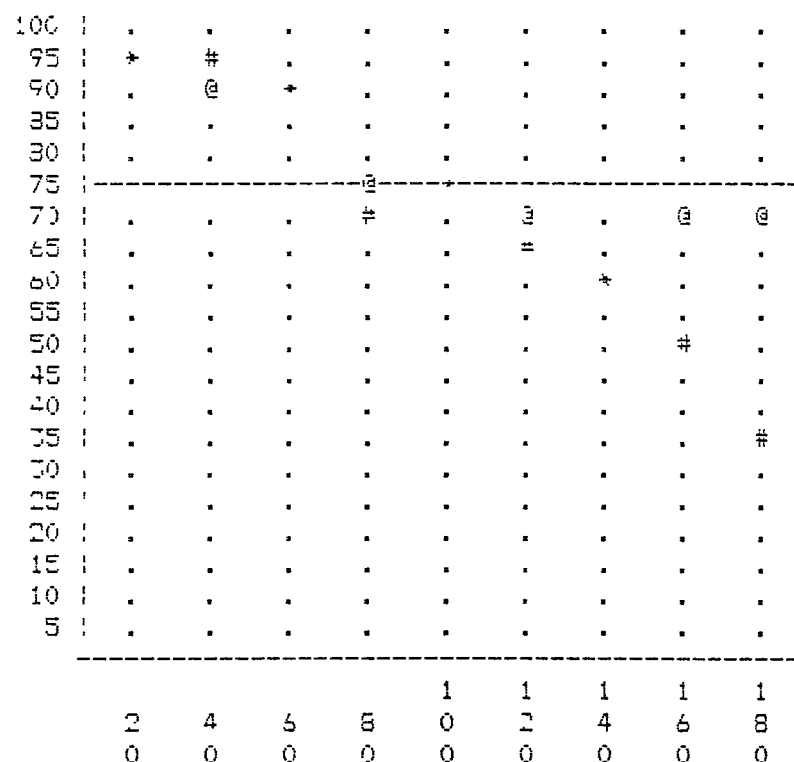
* - indicates no upper bound for confidence interval.

Graph of % of E_0/B_1 Estimates Falling in 75% CI

= B_0 @ = B_1 * = $B_0 \notin B_1$

Gamma₀ = 200 Phi = 0.0010 runs : 200

%good



Me	Mean			%good	B ₁ CI		E ₀ CI	E ₀	Mean	
	B ₁	B ₁ Min	B ₁ Max		%high	%low			E ₀ min	B ₀ max
320	0.001881	-0.007038	0.010800	0.970	0.030	0.000	0.970	2886.5	22.9	*
340	0.000989	-0.001972	0.003951	0.945	0.055	0.000	0.950	4825.7	51.0	*
360	0.000942	-0.000562	0.002446	0.915	0.085	0.000	0.910	4562.7	83.5	*
380	0.000863	-0.000048	0.001774	0.760	0.085	0.155	0.715	2669.0	116.2	*
100	0.000915	0.000313	0.001517	0.780	0.070	0.150	0.750	941.9	148.4	958.9
120	0.000938	0.000519	0.001357	0.730	0.100	0.170	0.685	271.7	172.6	931.2
140	0.000965	0.000665	0.001264	0.645	0.155	0.200	0.630	222.0	196.6	348.1
160	0.000949	0.000731	0.001166	0.710	0.090	0.200	0.545	215.7	195.5	258.7
180	0.000936	0.000780	0.001092	0.700	0.050	0.250	0.390	212.7	202.2	230.2

* - indicates no lower bound for confidence interval.

Graph of % of B_0/B_1 Estimates Falling in 80% CI

= B_0 @ = B_1 * => $B_0 \geq B_1$

Gamma₀ = 200 Phi = 0.0010 runs : 200

%good

100
95	*	+
90	.	.	+
85	.	.	.	@
80									
75	*	.	@	.
70	@	.	@
65	*	.	.
60	#	.
55
50
45
40	#
35
30
25
20
15
10
5

					1	1	1	1	1	
	2	4	6	8	0	2	4	6	8	# Failures Generated
	0	0	0	0	0	0	0	0	0	

Me	B_1	Mean B_1 Min	B_1 Max	%good	B_1 CI %high %low	B_0 CI %good	B_0	Mean B_0 min	B_0 max
020	0.001881	-0.008046	0.011808	0.980	0.020 0.000	0.980	2886.5	22.2	*
040	0.000989	-0.002307	0.004286	0.950	0.050 0.000	0.950	4825.7	48.8	*
060	0.000942	-0.000732	0.002616	0.925	0.075 0.000	0.920	4562.7	80.7	*
080	0.000863	-0.000151	0.001877	0.875	0.065 0.060	0.800	2669.0	111.9	*
100	0.000915	0.000245	0.001585	0.845	0.040 0.115	0.815	941.9	143.6	1145.7
120	0.000938	0.000471	0.001404	0.770	0.075 0.155	0.765	271.7	167.7	561.5
140	0.000965	0.000631	0.001298	0.745	0.110 0.145	0.675	222.0	183.7	387.3
160	0.000949	0.000706	0.001191	0.765	0.065 0.170	0.600	215.7	193.9	268.4
180	0.000936	0.000762	0.001109	0.725	0.040 0.235	0.425	212.7	201.3	232.9

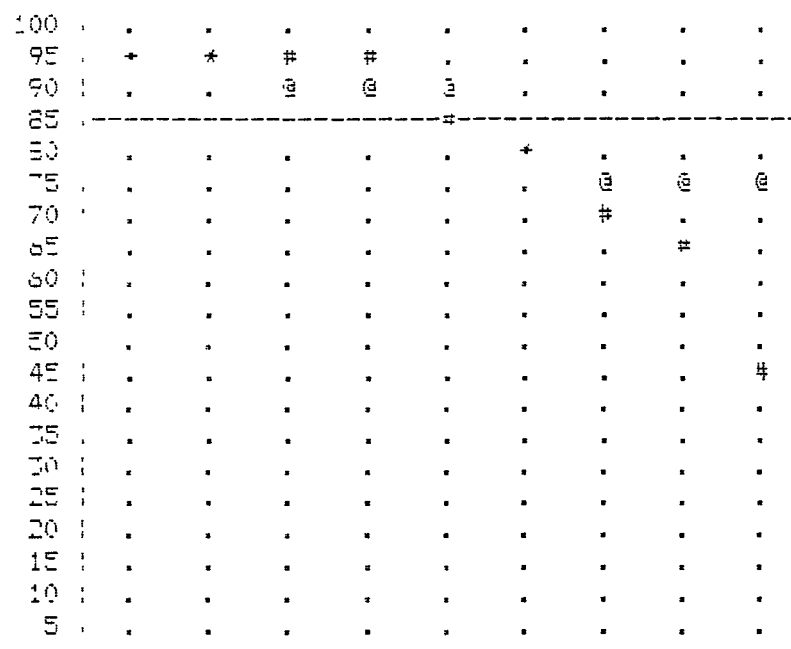
* - indicates no upper bound for confidence interval.

Graph of % of B_0/B_1 Estimates Falling in 85% CI

= B_0 @ = B_1 * = $B_0 \geq B_1$

Gamma₀ = 200 Phi = 0.0010 runs : 200

%good



2 4 6 8 10 12 14 16 18 20
0 0 0 0 0 0 0 0 0 0

= Failures Generated

Me	B_1	Mean B_1 Min	B_1 Max	%good	B_1 CI %high %low	B_0 CI %good	B_0	Mean B_0 min	B_0 max
020	0.001891	-0.009287	0.013049	0.980	0.020 0.000	0.980	2886.5	21.6	*
040	0.000989	-0.002719	0.004698	0.960	0.040 0.000	0.960	4825.7	47.2	*
060	0.000942	-0.000941	0.002825	0.945	0.055 0.000	0.950	4562.7	77.0	*
080	0.000863	-0.000278	0.002003	0.930	0.050 0.020	0.955	2669.0	107.8	*
100	0.000915	0.000161	0.001669	0.905	0.025 0.070	0.865	941.9	137.9	873.7
120	0.000978	0.000413	0.001463	0.820	0.055 0.125	0.805	271.7	162.9	627.3
140	0.000965	0.000590	0.001340	0.785	0.100 0.115	0.735	222.0	180.6	383.4
160	0.000949	0.000676	0.001221	0.790	0.050 0.160	0.670	215.7	192.0	289.2
180	0.000936	0.000740	0.001131	0.775	0.040 0.185	0.450	212.7	200.2	236.6

* - indicates no upper bound for confidence interval.

Graph of % of B_0/B_1 Estimates Falling in 90% CI

=, B_0 @ =, B_1 * =, $B_0 \& B_1$

Gamma₀ = 200 Fh1 = 0.0010 runs : 200

%good

100
95	*	*	*	*
90									
85	*	.	.	.
80	@	@	@
75	#	#	.
70
65
60
55
50	#
45
40
35
30
25
20
15
10
5

2 4 6 8 1 1 1 1 1
0 0 0 0 0 0 0 0 0

Failures Generated

Me	B_1	Mean B_1 Min	B_1 Max	%good	B_1 CI %high	%low	B_0 CI %good	B_0	Mean B_0 min	B_0 max
020	0.001881	-0.010877	0.014639	0.990	0.010	0.000	0.985	2886.5	21.3	*
040	0.000989	-0.003247	0.005226	0.965	0.035	0.000	0.965	4825.7	45.4	*
060	0.000942	-0.001209	0.003093	0.975	0.025	0.000	0.965	4562.7	73.2	*
080	0.000863	-0.000440	0.002166	0.975	0.025	0.000	0.970	2669.0	102.4	*
100	0.000915	0.000054	0.001776	0.940	0.020	0.040	0.905	941.9	132.7	1972.0
120	0.000938	0.000338	0.001537	0.865	0.040	0.095	0.850	271.7	157.7	887.1
140	0.000965	0.000536	0.001393	0.845	0.070	0.085	0.795	222.0	177.1	1200.8
160	0.000949	0.000637	0.001260	0.840	0.045	0.115	0.765	215.7	189.3	284.6
180	0.000936	0.000712	0.001159	0.830	0.020	0.150	0.520	212.7	198.9	241.8

* - indicates no upper bound for confidence interval.

Graph of % of B_0/B_1 Estimates Falling in 95% CI

= B_0 @ => B_1 * => $B_0 \& B_1$

Gamma₀ = 200 Phi = 0.0010 runs : 200

%good

100	*
95									
90	*	@	@	@
85	#	.	.
80	#	.
75
70
65
60
55	#
50
45
40
35
30
25
20
15
10
5

2	4	6	8	1	1	1	1	1	
0	0	0	0	0	0	0	0	0	# Failures Generated

Me	B_1	Mean B_1 Min	B_1 Max	%good	B_1 CI %high %low	B_0 CI %good	B_0	Mean B_0 min	B_0 max
620	0.001881	-0.013320	0.017082	1.000	0.000 0.000	1.000	2886.5	0.0	*
640	0.000989	-0.004058	0.006037	0.995	0.005 0.000	0.995	4825.7	44.1	*
660	0.000942	-0.001621	0.003505	0.990	0.010 0.000	0.995	4562.7	69.5	*
680	0.000863	-0.000690	0.002415	0.980	0.020 0.000	0.980	2669.0	96.9	*
100	0.000915	-0.000111	0.001941	0.975	0.020 0.005	0.955	941.9	125.3	*
120	0.000938	0.000223	0.001652	0.940	0.025 0.035	0.930	271.7	150.6	1122.7
140	0.000965	0.000454	0.001475	0.920	0.030 0.050	0.870	222.0	171.7	936.5
160	0.000949	0.000578	0.001320	0.915	0.020 0.065	0.830	215.7	186.3	320.0
180	0.000936	0.000670	0.001201	0.915	0.000 0.085	0.585	212.7	197.1	251.7

* - indicates no upper bound for confidence interval.

Appendix F - Mathematics

APPENDIX F INDEX

F-1	Equations Defining Basic Model
F-3	Confidence Interval Equations
F-4	Failure Time Generation Equations

EQUATIONS DEFINING BASIC MODEL

1. The Failure Intensity Equation of the model is given by:

$$\lambda(t) = \lambda_0 \exp\left[-\frac{\lambda_0}{\gamma_0} t\right]$$

where t = time, λ_0 = estimated initial failure intensity
and γ_0 = estimated total number of software faults.

2. For generality, the Failure Intensity Equation is reparameterized as:

$$\lambda(t) = \beta_0 \beta_1 \exp(-\beta_1 t)$$

where $\beta_0 = \gamma_0$, estimated total failures, and

$$\beta_1 = \frac{\lambda_0}{\gamma_0}, \text{ estimated failure rate per fault.}$$

3. The Likelihood Equation is given by:

$$L(\beta) = \left[\prod_{i=1}^{m_e} f(t_i | t_{i-1}) \right] P[T_{m_e+1} > t_e | T_{m_e} = t_{m_e}]$$

where m_e equals the no. of faults found/fixed.

t_{m_e} equals time of last fault.

t_i equals total time of i th failure.

t_e equals time at end of testing.

T_i is the random variable for time of i th failure.

$f(t_i | t_{i-1})$ is the conditional density function of (T_1, \dots, T_{m_e})

$P[T_{m_e+1} > t_e | T_{m_e} = t_{m_e}]$ is the conditional distribution of T_{m_e+1}

The Conditional Density Function is given by :

$$f(t_i | t_{i-1}) = \lambda(t_i) \exp\{-[\mu(t_i) - \mu(t_{i-1})]\}$$

where the Mean Value Function (mean faults by time t) is given by:

$$\mu(t) = \beta_0 [1 - \exp(-\beta_1 t)]$$

The Conditional Distribution is given by:

$$P[T_{m_e+1} > t_e | T_{m_e} = t_{m_e}] = \exp\{-[\mu(t_e) - \mu(t_{m_e})]\}$$

4. Substitution into the Likelihood Equation yields:

$$L(\beta) = \left[\prod_{i=1}^{m_e} \lambda(t_i) \right] \exp[-\mu(t_e)]$$

5. The logarithm of the Likelihood Equation is:

$$\ln L(\beta) = \sum_{i=1}^{m_e} \ln \lambda(t_i) - \mu(t_e)$$

6. Thus, the Maximum Likelihood Equation becomes:

$$\sum_{i=1}^{m_o} \frac{1}{\lambda(t_i)} \frac{\partial \lambda(t_i)}{\partial \beta_k} - \frac{\partial \mu(t_o)}{\partial \beta_k} = 0, \quad k = 0, \dots, w$$

7. It can be verified by direct substitution into the previous equation with $k = 0$ that all scaled Mean Value Functions yield the result:

$$\mu(t_o; \beta) = m_o$$

which can be rewritten as

$$\beta_o = \frac{m_o}{1 - \exp(\beta_1 t_o)}$$

8.

The probability observing m_o failures by time t_o is given by

$$P[M(t_o) = m_o] = \frac{[\mu(t_o)]^{m_o}}{m_o!} \exp[-\mu(t_o)]$$

9. After appropriate substitutions, the Conditional Likelihood Equation becomes:

$$L(\beta_A | m_o) = \frac{m_o! \prod_{i=1}^{m_o} \lambda(t_i)}{[\mu(t_o)]^{m_o}}$$

10.

The Conditional Likelihood of β_1 given m_o failures is:

$$L(\beta_1 | m_o) = \frac{m_o! \beta_1^{m_o} \exp(-\beta_1 \sum_{i=1}^{m_o} t_i)}{[1 - \exp(-\beta_1 t_o)]^{m_o}}$$

11.

Using the ln of the maximum conditional likelihood eq., the point estimate for β_1 is given as:

$$\frac{m_o}{\beta_1} - \frac{m_o t_o}{\exp(\beta_1 t_o) - 1} - \sum_{i=1}^{m_o} t_i = 0$$

Confidence Interval Equations

12.

The MLE of β_k is asymptotically normally distributed with mean β_k , variance $\frac{1}{I(\beta_k)}$, where $I(\beta_k)$ is the expected, Fisher information given by:

$$I(\beta_k) = E \left[-\frac{\partial^2 \ln L(\beta_k; Y_D)}{\partial \beta_k^2} \right]$$

13. Thus we can write,

$$\frac{\beta_k - \hat{\beta}_k}{\sqrt{\frac{1}{I(\beta_k)}}} \sim N(0, 1)$$

14.

The upper/lower limits of an approximate 100(1- α) percent Confidence Interval for β_k are given by

$$\hat{\beta}_k \pm \frac{K_{1-\alpha/2}}{\sqrt{I(\hat{\beta}_k)}}$$

where $K_{1-\alpha/2}$ is the appropriate normal deviate.

15.

$$I(\beta_1) = m_\theta \left[\frac{1}{\beta_1^2} - \frac{t_\theta^2 \exp(\beta_1 t_\theta)}{[\exp(\beta_1 t_\theta) - 1]^2} \right]$$

Substituting the confidence interval values for B_1 into Equation 7, a corresponding confidence interval is obtained for B_0 .

Time Generation Equations

16. The Distribution of Faults as a function of time is given as:

$$F(t) = 1 - e^{-\int_{tsum}^{t+tsum} \lambda(x) dx}$$

The simplification of the integral is found to be:

$$\begin{aligned} \int_{tsum}^{t+tsum} \lambda(x) dx &= \int_{tsum}^{t+tsum} \lambda_0 e^{(\lambda_0/\gamma_0)x} dx \\ &= \left[-\gamma_0 e^{-(\lambda_0/\gamma_0)x} + C \right]_{tsum}^{t+tsum} \\ &= \left[-\gamma_0 e^{-(\lambda_0/\gamma_0)(t+tsum)} + C - (-\gamma_0 e^{-(\lambda_0/\gamma_0)tsum} + C) \right] \\ &= \gamma_0 e^{-(\lambda_0/\gamma_0)tsum} [1 - e^{-(\lambda_0/\gamma_0)t}] \end{aligned}$$

17. Substituting the integrated exponent we get:

$$F(t) = 1 - e^{-\gamma_0 e^{-(\lambda_0/\gamma_0)tsum} (1 - e^{-(\lambda_0/\gamma_0)t})}$$

18. Randomly generate values for $F(t)$, r , and solve for t .

$$1 - r = e^{-\gamma_0 e^{-(\lambda_0/\gamma_0)tsum} (1 - e^{-(\lambda_0/\gamma_0)t})}$$

19. Take the natural logarithm of both sides and simplify.

$$\ln(1 - r) = -\gamma_0 e^{-(\lambda_0/\gamma_0)tsum} (1 - e^{-(\lambda_0/\gamma_0)t})$$

20. Rearrange algebraically.

$$\frac{-\ln(1 - r)}{\gamma_0 e^{-(\lambda_0/\gamma_0)tsum}} = 1 - e^{-(\lambda_0/\gamma_0)t}$$

21. Rearrange algebraically.

$$1 + \frac{\ln(1 - r)}{\gamma_0 e^{-(\lambda_0/\gamma_0)tsum}} = e^{-(\lambda_0/\gamma_0)t}$$

22. Take the natural logarithm of both sides.

$$\ln \left(1 + \frac{\ln(1 - r)}{\gamma_0 e^{-(\lambda_0/\gamma_0)tsum}} \right) = -(\lambda_0/\gamma_0)t$$

23. Rearrange algebraically.

$$t = \frac{-\gamma_0}{\lambda_0} \ln \left(1 + \frac{\ln(1 - r)}{\gamma_0 e^{-(\lambda_0/\gamma_0)tsum}} \right)$$

Appendix G - Code

This Appendix contains the source code of programs used to calculate failure times, parameter estimates, confidence intervals, and produce graphs used.

APPENDIX G INDEX

G-1	Basic.p : Failure Times and Estimation
G-6	Confidence.p : Confidence Intervals
G-17	Graph.p : Summary Graphs
G-22	Histogram.p : Scatter Plots

This program generates failure data using the distribution of the Basic Execution Time Model. It then calculates estimates for the reliability parameters B_0 and B_1 and writes these to an output file. The program reads input from a text file. Inputs are the total program faults (γ_0), failure intensity per fault (ϕ_1), the total number of faults to generate (Me), the number of runs to perform for each failure set size ($runs$), and the number of data repetitions ($repeats$). The output of this program is used as input to the Histogram and Summary graph programs.

{ Programmer : Keith J. Lirette

Implementation of Musa - Basic Execution Time Model }

PROGRAM BASIC (INPUT, OUTPUT, in_file, out_file);

CONST

epsilon = 0.01;
max_ints = 200; { max: number of time intervals }

TYPE

array_type = ARRAY[1..max_ints] of REAL;
matrix_type = ARRAY[1..10, 1..max_ints] of REAL;

VAR

Me, { number of bugs removed }
gamma_0, { number of total bugs }
repeats, { number of data generations per run }
J,
l,
k, { loop counters }
runs, { number of runs desired }
start, { number of errors to 1st generate }
Me_offset, { amt to increase # errors generated by }
num_pts, { number of different # errors wanted }
num_errors : INTEGER;

temp_sum, { dummy variable }
phi : REAL; { failure rate per bug, input }
b0_est, { estimate of b_0 }
b1_est, { estimate of b_1 }
te, { time to end of testing }
time_sum : matrix_type;
t : array_type; { total time to failures }
in_file,
out_file : TEXT;

{-----}

FUNCTION get_b0_est(Me : INTEGER;
te,
b1_est : REAL) : REAL;

{ function to get estimate of b_0 given b_1_est }

begin
get_b0_est := Me / (1 - exp(-b1_est * te));
end;

```

{-----}
FUNCTION calc_b1_est(Me      : INTEGER;      { # errors found }
                    te,      { time to end of testing }
                    time_sum : REAL) : REAL; { sum of failure times }

    { function to get estimate of b1 given failure data }

VAR
    b1_est,      { holds working estimate of b1 }
    low,         { low end of estimation interval }
    high,        { high end of estimation interval }
    lhs_est,     { var to hold lhs of eq value }
    rhs_est,     { var to hold rhs of eq value }
    Me_x_te      : REAL; { var to hold product calculated once }

{-----}

FUNCTION lhs(Me      : INTEGER) : REAL;

    { function to get value of left hand side of equation solving
      for b1 }

begin
    lhs := Me;
end; { lhs }

{-----}

FUNCTION rhs(Me_x_te,
            te,
            time_sum,
            b1_est : REAL) : REAL;

    { function to get value of right hand side of equation solving
      for b1 }

begin
    rhs := b1_est * ( Me_x_te / (exp(b1_est * te) - 1) + time_sum);
end;

{-----}
begin
    { initial guess at b1 to start calculations }
    b1_est := 0.0001;

    { calculate product once and store for use }
    Me_x_te := Me * te;

    { get left hand side, right hand side guess values }
    lhs_est := lhs (Me);
    rhs_est := rhs (Me_x_te, te, time_sum, b1_est);

    { if lhs_est > rhs_est increase b1_est until not true }
    while lhs_est > rhs_est do
    begin
        b1_est := b1_est + 0.0001;
        lhs_est := lhs (Me);
        rhs_est := rhs (Me_x_te, te, time_sum, b1_est);
    end;

```

```

low := b1_est - 0.0001;
high := b1_est;

{ perform bisection until value at which relationship first
  changes is found
}
repeat
  b1_est := (low + high) / 2;
  lhs_est := lhs (Me);
  rhs_est := rhs (Me_x_te, te, time_sum, b1_est);
  if lhs_est < rhs_est then
    high := b1_est
  else
    low := b1_est;
until abs(lhs_est - rhs_est) <= epsilon;

calc_b1_est := b1_est;
end;
```

```

{-----}
{ procedure to generate a set of failure times using number of repeats
  indicated.
}
procedure new_data (      gamma_0,
                        Me,
                        repeats : INTEGER;
                        var time_sum : REAL;
                        phi : REAL;
                        var      t : array_type);

var
  j      : INTEGER;      { indices }
previous : REAL;         { value of previous time }
t_avg    : array_type;   { array to hold running total to be averaged }
bust     : boolean;      { indicates overflow condition }

begin
  { initialize array that will hold running totals of times }
  for j := 1 to Me do
    t_avg[j] := 0.0;

  j := 0;

  { get number of time samples indicated by variable repeats }
  repeat
    { initialize variables }
    bust := false;
    previous := 0.0;
    i := 0;

    { generate required times }
    repeat
      i := i + 1;

      { test for overflow condition }
      if (previous*phi > 719.0) then
        begin
          bust := true;
        end
      end
    until bust;
  until bust;
end;
```

```

else { generate next time interval }
begin
  t[i] := previous + (1.0/phi) *
    ln ( 1 - ln(1-random(1.0)) / (gamma_0 * exp (-phi * previous) ) );
  previous := t[i];
end;
until (i = Me) or bust; { until all times gotten or overflow }

{ add time intervals to t_avg to keep running totals }
if not bust then
begin
  for i := 1 to Me do
    t_avg[i] := t_avg[i] + t[i];
  j := j + 1;
end;

until j = repeats; { repeat until required samples performed }

{ calculate average times and total of time averages }
time_sum := 0.0;
for j := 1 to Me do
begin
  t[j] := t_avg[j] / repeats;
  time_sum := time_sum + t[j];
end;
end;

```

```

-----}

BEGIN
  { prepare input and output file }
  reset(in_file);
  rewrite(out_file);

  while not eof(in_file) do
  begin
    { get parameters to produce output for }
    readln(in_file, gamma_0, phi, Me, repeats, runs);

    { start point is 10% of total number of errors wanted }
    start := round( (gamma_0 * 0.1) );

    { generate 10% more errors until total number of errors generated }
    Me_offset := round( (gamma_0 * 0.1) );

    { calculate total number of error sets }
    num_pts := (Me - Start) div Me_offset + 1;

    { produce fault data for number of runs wanted }
    for j := 1 to runs do
    begin
      new_data(gamma_0, Me, repeats, temp_sum, phi, t);

      { use subset of times generated to get estimates using 10, 20...% }
      for k := 1 to num_pts do
      begin
        { calculate number of error times to use }
        num_errors := start + (k-1) * Me_offset;
        { calculate time sum for errors used }
        time_sum[k,j] := 0.0;
        for l := 1 to num_errors do
          time_sum[k,j] := time_sum[k,j] + t[l];

        { set value for total testing time }
        te[k,j] := t[num_errors];

        { get estimates for b1 and b0 }
        b1_est[k,j] := calc_b1_est (num_errors, te[k,j], time_sum[k,j]);
        b0_est[k,j] := get_b0_est(num_errors, te[k,j], b1_est[k,j]);
      end;
    end;

    { write out set of calculated values }
    for k := 1 to num_pts do
    begin
      num_errors := start + (k-1) * Me_offset;
      writeln(out_file, gamma_0, ' ', phi, ' ', num_errors, ' ', repeats, ' ', runs);
      for j := 1 to runs do
        writeln(out_file, b1_est[k,j], b0_est[k,j], time_sum[k,j], te[k,j]);
      end;
    end;
  end;
END.

```

This program produces Confidence Interval summary graphs. The program reads input from a text file. Inputs are the total program faults (γ_0), failure intensity per fault (ϕ_1), the total number of faults to generate (M_e), and the number of runs to perform for each failure set size (runs). The program produces the desired number of runs starting with 10% of the total program faults, increases by 10% until M_e fault trials are runs. Confidence intervals (50, 55,...,95%) are calculated for each of the trial sizes generated and summary graphs are plotted.

{ Programmer : Keith J. Lirette

PROGRAM CONFIDENCE(INPUT, OUTPUT, in_gph, out_gph);

CONST

```
epsilon = 0.01;
epsilon2 = 0.00000001;
max_ints = 200;           { max number of time intervals }
num_c1 = 10;              { number of confidence intervals }
line_length = 80;
mark1 = '#';
mark2 = '@';
combo_mark = '*';
num_grps = 20;           { max number of percentage divisions }
```

TYPE

```
array_type = ARRAY[1..max_ints] of REAL;
matrix_type = ARRAY[1..10, 1..max_ints] of REAL;
line_type = ARRAY[1..line_length] of char;
```

k_rec_type = record

```
    c1 : integer;
    val : real;
end;
```

k_arr_type = ARRAY[1..num_c1] of k_rec_type;

grp_rec_type = record

```
    value : real;
    labell : integer;
end;
```

data_rec_type = record

```
    b0_in_pct,
    b1_in_pct,
    b1_low_mean,
    b1_high_mean,
    b0_mean,
    b1_mean,
    b1_high,
    b1_low,
    b0_min_mean,
    b0_max_mean : real;
    labell : line_type;
    used_1,
    used_2 : boolean;
end;
```

grp_array_type = array[1..num_grps] of grp_rec_type;

data_array_type = array[1..num_grps] of data_rec_type;

VAR

```

Me,                                { number of bugs removed }
gamma_0      : INTEGER;           { number of total bugs }
time_sum,    { sum of all failure times }
phi,         { failure rate per bug, input }
I_val,       { fisher information of b1_est }
b1_offset,   { magnitude of confidence interval around b1 }
b0_min,      { lower value of confidence interval }
b0_max,      { upper value of confidence interval }
b1_min,      { lower value of confidence interval }
b1_max,      { upper value of confidence interval }
b1_mean,     { average of b1 over number of runs }
b1_min_sum,  { sum variable used to calc b1 min mean }
b1_max_sum,  { sum variable used to calc b1 max mean }
b0_min_sum,  { sum variable used to calc b0 min mean }
b0_max_sum,  { sum variable used to calc b0 max mean }
b1_sum,      { sum variable used to calc b1 mean }
b0_sum      : REAL;              { sum variable used to calc b0 mean }

repeats,    { number of data generations per run }
i1,
j,
b1_in_cnt,  { # of b1 ests in own conf interval }
b0_in_cnt,  { # of b0 ests in own conf interval }
b0_untest,  { # of b0 with no upper bound }
b0_min_lower, { # of b0 with no upper bound but
               whose lower bound is < estimate }
b1_high,    { # b1 ests above own conf interval }
b1_low,     { # b1 ests below own conf interval }
runs,       { number of runs desired }
start,      { number of errors to generate first }
Me_offset,  { amt to increase number of errors by }
num_pts,    { number of different # of errors }
l,          { counter }
num_errors, { number of errors generated }
k           : INTEGER;           { counter }
k_val      : k_arr_type;         { holds confidence intervals and
                                   corresponding k values }

b0_est,     { estimate of b0 }
b1_est,     { estimate of b1 }
te         : matrix_type;        { time to end of testing }

pos_array   : grp_array_type;    { holds % divisions for graph and
                                   labels for y axis }

data        : data_array_type;   { holds calculated fault data and
                                   graph labels }
t           : array_type;        { total time to failures }

line        : line_type;         { graph line built to print }
in_gph,     { text file of input parameters }
out_gph     : TEXT;              { output file for generated graphs }

```

```

{-----}
FUNCTION I (Me      : INTEGER;
           b1_est,
           te       : REAL) : REAL;

    { function to get fisher information for b1 }

begin

    I := Me * ( 1 / SQR(b1_est) -
                SQR(te) * exp(b1_est * te) / SQR( exp(b1_est * te) - 1) );

end;

{-----}
FUNCTION get_b0_est(Me      : INTEGER;
                   te,
                   b1_est   : REAL) : REAL;

    { function to get estimate of b0 given b1_est }

begin

    get_b0_est := Me / ( 1 - exp(-b1_est * te) );

end;

{-----}
FUNCTION calc_b1_est(Me      : INTEGER;           { # errors found }
                    te,      { time to end of testing }
                    time_sum : REAL) : REAL;      { sum of failure times }

    { function to get estimate of b1 given failure data }

VAR
    b1_est,           { holds working estimate of b1 }
    low,              { low end of estimation interval }
    high,             { high end of estimation interval }
    lhs_est,          { var to hold lhs of eq value }
    rhs_est,          { var to hold rhs of eq value }
    Me_x_te           : REAL; { var to hold product calculated once }

{-----}
FUNCTION lhs(Me      : INTEGER) : REAL;

    { function to get value of left hand side of equation solving
      for b1 }

begin

    lhs := Me;

end; { lhs }

{-----}
FUNCTION rhs(Me_x_te,
            te,
            time_sum,
            b1_est   : REAL) : REAL;

    { function to get value of right hand side of equation solving
      for b1 }

begin

    rhs := b1_est * ( Me_x_te / (exp(b1_est * te) - 1) + time_sum);

end;

```



```

{-----}
begin
  { initial guess at b1 to start calculations }
  b1_est := 0.0001;

  { calculate product once and store for use }
  Me_x_te := Me * te;

  { get left hand side, right hand side guess values }
  lhs_est := lhs (Me);
  rhs_est := rhs (Me_x_te, te, time_sum, b1_est);

  { if lhs_est > rhs_est increase b1_est until not true }
  while lhs_est > rhs_est do
  begin
    b1_est := b1_est + 0.0001;
    lhs_est := lhs (Me);
    rhs_est := rhs (Me_x_te, te, time_sum, b1_est);
  end;
  low := b1_est - 0.0001;
  high := b1_est;

  { perform bisection until value at which relationship first
    changes is found
  }
  repeat
    b1_est := (low + high) / 2;
    lhs_est := lhs (Me);
    rhs_est := rhs (Me_x_te, te, time_sum, b1_est);
    if lhs_est < rhs_est then
      high := b1_est
    else
      low := b1_est;
  until abs(lhs_est - rhs_est) <= epsilon;
  calc_b1_est := b1_est;
end;

```

```

-----}
procedure new_data (      gamma_0,
                        Me  : INTEGER;
                        var time_sum : REAL;
                        phi  : REAL;
                        var      t : array_type);
var
    i      : INTEGER;      { index }
    previous : REAL;        { value of previous time generated }
    busted   : boolean;     { indicates if overflow condition present }
begin
    { repeat until valid times generated }
    repeat
        { initialize values }
        previous := 0.0;
        time_sum := 0.0;

        busted := false;
        i := 0;

        { generate times until done or overflow condition }
        while (i < Me) and not busted do
            begin
                i := i + 1;

                { calculate new time }
                t[i] := previous + (1.0/phi) *
                    ln ( 1 - ln(1-random(1.0)) / (gamma_0 * exp (-phi * previous) ) );
                previous := t[i];

                { add new time value to running total }
                time_sum := time_sum + t[i];

                { test for overflow condition }
                busted := ( phi * previous) > 88.0;
            end;
        until not busted;
    end;
    -----}
    { set confidence interval sizes and corresponding k values }
    procedure initialize_k_val (var k_val : k_arr_type);
    begin
        k_val[1].ci := 50;
        k_val[1].val := 0.675;
        k_val[2].ci := 55;
        k_val[2].val := 0.755;
        k_val[3].ci := 60;
        k_val[3].val := 0.84;
        k_val[4].ci := 65;
        k_val[4].val := 0.935;
        k_val[5].ci := 70;
        k_val[5].val := 1.035;
        k_val[6].ci := 75;
        k_val[6].val := 1.15;
        k_val[7].ci := 80;
    end;
end;

```

```

k_val[7].val := 1.28;
k_val[8].c1 := 85;
k_val[8].val := 1.44;
k_val[9].c1 := 90;
k_val[9].val := 1.645;
k_val[10].c1 := 95;
k_val[10].val := 1.96;
end;

```

```

{-----}
{ initialize array to hold % group break out and y-axis labels }
procedure initial_pos_array (var pos_array : grp_array_type);

```

```

var i : integer;

```

```

begin

```

```

    pos_array[1].value := 0.999;

```

```

    pos_array[1].label1 := 100;

```

```

    for i := 2 to num_grps do

```

```

        begin

```

```

            pos_array[i].value := pos_array[i-1].value - 0.05;

```

```

            pos_array[i].label1 := pos_array[i-1].label1 - 5;

```

```

        end;

```

```

    end;

```

```

{-----}
{ procedure to convert integer < 1000 to a string }
procedure strx (tag : integer; var val_str : line_type);

```

```

var temp : integer;

```

```

begin

```

```

    temp := tag div 100;

```

```

    val_str[1] := chr(48+temp);

```

```

    tag := tag - temp*100;

```

```

    temp := tag div 10;

```

```

    val_str[2] := chr(48+temp);

```

```

    tag := tag - temp*10;

```

```

    temp := tag;

```

```

    val_str[3] := chr(48+temp);

```

```

end;

```

```

{-----}
{ procedure to get data line to print on graph }
procedure get_line( pos_array : grp_array_type;
                   var data : data_array_type;
                   grp_num,
                   data_pts : integer;
                   highlight : boolean);

var
    mark_pos : INTEGER;
    line      : array[1..50] of char;
    fill      : CHAR;
begin
    { highlight indicates this line is the line matching the confidence
      interval size, so mark it with dashes }
    if highlight then
        fill := '-';
    else
        fill := ' ';

    { initialize line }
    for i := 1 to data_pts*5 do
        line[i] := fill;

    { if line not on confidence interval size, set dots every 5 so
      reading is easier }
    if not highlight then
        for i := 1 to data_pts do
            line[i*5 - 2] := '.';

    write(out_gph, pos_array[grp_num].label1:4, ' ');

    { determine if any data points fall in indicated % group and mark }
    for i := 1 to data_pts do
        begin
            mark_pos := i*5 - 2;

            if (pos_array[grp_num].value < data[i].b0_in_pct) then

                if not data[i].used_1 then
                    begin
                        line[mark_pos] := mark1;
                        data[i].used_1 := true;
                    end;

            if (pos_array[grp_num].value < data[i].b1_in_pct) then

                if not data[i].used_2 then
                    begin
                        if line[mark_pos] = mark1 then
                            line[mark_pos] := combo_mark
                        else
                            line[mark_pos] := mark2;
                        data[i].used_2 := true;
                    end;
                end;

        end;

end;

```

```
{ print line to output file }
for i := 1 to data_pts*5 do
```

```
    write(out_gph, line[i]);
    writeln(out_gph);
end;
```

```
-----}
```

```
BEGIN
```

```
reset(in_gph);
rewrite(out_gph);
```

```
initial_pos_array(pos_array);
initialize_k_val(k_val);
```

```
while not eof(in_gph) do
begin
```

```
    readln(in_gph, gamma_0, phi, Me, runs);
```

```
    { determine number of errors to first generate }
```

```
    start := round( (gamma_0 * 0.1) );
```

```
    { determine amt to increase errors by }
```

```
    Me_offset := round( (gamma_0 * 0.1) );
```

```
    { determine number of different errors set sizes }
```

```
    num_pts := (Me - Start) div Me_offset + 1;
```

```
    { generate data for number of desired runs -- total errors }
```

```
    for j := 1 to runs do
```

```
    begin
```

```
        new_data(gamma_0, Me, time_sum, phi, t);
```

```
        { for each error set size, use # of errors from those errors
          generated and calculate b0 and b1 estimates }
```

```
        for k := 1 to num_pts do
```

```
        begin
```

```
            num_errors := start + (k-1) * Me_offset;
```

```
            time_sum := 0.0;
```

```
            for l := 1 to num_errors do
```

```
                time_sum := time_sum + t[l];
```

```
            te[k, j] := t[num_errors];
```

```
            b1_est[k, j] := calc_b1_est (num_errors, te[k, j], time_sum);
```

```
            b0_est[k, j] := get_b0_est(num_errors, te[k, j], b1_est[k, j]);
```

```
        end;
```

```
    end;
```

```
    { calculate averages for generated estimates }
```

```
    for k := 1 to num_pts do
```

```
    begin
```

```
        b1_sum := 0.0;
```

```
        b0_sum := 0.0;
```

```
        for j := 1 to runs do
```

```
        begin
```

```
            b1_sum := b1_sum + b1_est[k, j];
```

```
            b0_sum := b0_sum + b0_est[k, j];
```

```
        end;
```

```
        data[k].b1_mean := b1_sum / runs;
```

```
        data[k].b0_mean := b0_sum / runs;
```

```
    end;
```

```

{ test data on different cc intervals in k_val }
for ii := 1 to num_ci do
begin
  { for each error set size, generate conf intervals on values }
  for k := 1 to num_pts do
  begin
    num_errors := start + (k-1) * Me_offset;
    b0_in_cnt := 0;
    b0_untest := 0;
    b0_min_lower := 0;
    b1_in_cnt := 0;
    b1_high := 0;
    b1_low := 0;
    b1_min_sum := 0.0;
    b1_max_sum := 0.0;
    b0_min_sum := 0.0;
    b0_max_sum := 0.0;

    { for each data run, calculate confidence and test }
    for j := 1 to runs do
    begin
      I_val := I (num_errors, b1_est[k,j], te[k,j]);
      b1_offset := k_val[ii].val / SQRT(I_val);
      b1_min := b1_est[k, j] - b1_offset;
      b1_max := b1_est[k, j] + b1_offset;
      b1_min_sum := b1_min_sum + b1_min;
      b1_max_sum := b1_max_sum + b1_max;
      if b1_min <= epsilon2 then
      begin
        b1_min := 0.0;
        b0_untest := b0_untest + 1;
        if get_b0_est(num_errors, te[k, j], b1_max) < gamma_0 then
          b0_min_lower := b0_min_lower + 1;
        end
      else
      begin
        b0_min := get_b0_est(num_errors, te[k, j], b1_max);
        b0_min_sum := b0_min_sum + b0_min;
        b0_max := get_b0_est(num_errors, te[k, j], b1_min);
        b0_max_sum := b0_max_sum + b0_max;

        if (gamma_0 >= b0_min) and
           (gamma_0 <= b0_max) then
          b0_in_cnt := b0_in_cnt + 1;
        end;

        if phi < b1_min then
          b1_high := b1_high + 1
        else if phi > b1_max then
          b1_low := b1_low + 1
        else
          b1_in_cnt := b1_in_cnt + 1;
      end;

    end;

    strx(num_errors, data[k].labell);
    data[k].b0_in_pct := (b0_in_cnt + b0_min_lower)/runs;
    data[k].b1_in_pct := b1_in_cnt / runs;
  end;
end;

```

```

data[k].used_1 := false;
data[k].used_2 := false;
data[k].b1_high := b1_high / runs;
data[k].b1_low := b1_low / runs;

data[k].b1_low_mean := b1_min_sum / runs;
data[k].b1_high_mean := b1_max_sum / runs;

if b0_untest < runs then
begin
  data[k].b0_min_mean := b0_min_sum / (runs - b0_untest);
  if b0_untest >= (runs div 2) then
    data[k].b0_max_mean := 9999.0
  else
    data[k].b0_max_mean := b0_max_sum / (runs - b0_untest);
  end
else
begin
  data[k].b0_min_mean := 0.0;
  data[k].b0_max_mean := 9999.0;
end;
end;

( print results for one conf interval size )
writeln(out_gph, '      Graph of % of b0/b1 Estimates Falling in ',
  k_val[11].c1:3, '% CI ');
writeln(out_gph);
writeln(out_gph, '      # => b0      @ => b1      * => b0 & b1');
writeln(out_gph);
writeln(out_gph, ' Gamma_0 = ', gamma_0:3, '   Ph1 = ', ph1:5:4, ' ':5,
  'runs : ', runs:2);
writeln(out_gph);

writeln(out_gph);
writeln(out_gph, ' % in');
writeln(out_gph);
for j := 1 to num_grps do
begin
  get_line(pos_array, data, j, num_pts, k_val[11].c1 = pos_array[j].label1);
end;

write(out_gph, ' ':5, '-');
for k := 1 to num_pts do
  write(out_gph, '-----');
writeln(out_gph);

for l := 1 to 3 do
begin
  write(out_gph, ' ':6);
  for k := 1 to num_pts do
    write(out_gph, ' ', data[k].label1[11], ' ');
  if l = 2 then
    writeln(out_gph, ' # faults fixed')
  else
    writeln(out_gph);
end;
writeln(out_gph);
writeln(out_gph, '
Mean
%b1 ',

```

```

      '      %b0      Mean' );
writeln(out_gph, ' Me      b1      b1 Min      b1 Max      in c1      high',
      '      low      in c1      b0      b0 min      b0 max');
writeln(out_gph, '-----',
      '-----');

```

```

for k := 1 to num_pts do
  writeln(out_gph, data[k].label1, ' ', data[k].b1_mean:7:6, ' ',
    data[k].b1_low_mean:7:6, ' ', data[k].b1_high_mean:7:6, ' ',
    data[k].b1_in_pct:4:3, ' ',
    data[k].b1_high:4:3, ' ', data[k].b1_low:4:3, ' ',
    data[k].b0_in_pct:4:3, ' ', data[k].b0_mean:6:1, ' ',
    data[k].b0_min_mean:6:1, ' ', data[k].b0_max_mean:8:1);

```

```

  writeln(out_gph, chr(12));

```

```

end;

```

```

end;

```

```

close(in_gph);

```

```

close(out_gph);

```

```

END.

```


This program performs no data generation. It produces summary graphs of the of data generated by the BASIC.P program.

```
PROGRAM GRAPH;
```

```
const
  line_length = 65;
  mark = '*';
  num_grps = 31;           { number horizontal graph lines }
  max_pts = 20;           { max number of vertical graph lines }
```

```
type
  array_type = array[1..100] of REAL;
  line_type = STRING[line_length];
```

```
grp_rec_type = record
  value : real;
  labell : integer;
end;
```

```
data_rec_type = record
  value,
  b0_mean,
  b0_std,
  b1_mean,
  b1_std,
  tsum_mean,
  te_mean : real;
  labell : string[3];
  used : boolean;
end;
```

```
grp_array_type = array[1..num_grps] of grp_rec_type;
```

```
data_array_type = array[1..max_pts] of data_rec_type;
```

```
var
  gamma_0,
  repeats,
  runs,
  Me,
  l,
  k,
  temp_length,
  j : INTEGER;
  ph1 : REAL;
  pos_array : grp_array_type;
  data : data_array_type;
  b1,
  te,
  tsum,
  b0 : array_type;
  line : line_type;
  in_file : TEXT;
  out_file : TEXT;
  file_name,
  file_name2 : STRING[14];
  temp_str : STRING[3];
```

```

{-----}
{ This array holds the break out percentages for graph lines and the
  corresponding y-axis labels
}
procedure initial_pos_array (var pos_array : grp_array_type);

begin
  pos_array[1].value := 1.95;
  pos_array[1].label1 := 100;
  for i := 2 to 10 do
    begin
      pos_array[i].value := pos_array[i-1].value - 0.1;
      pos_array[i].label1 := pos_array[i-1].label1 - 10;
    end;
  pos_array[11].value := 1.045;
  pos_array[11].label1 := 5;
  for i := 12 to 21 do
    begin
      pos_array[i].value := pos_array[i-1].value - 0.01;
      pos_array[i].label1 := pos_array[i-1].label1 - 1;
    end;
  pos_array[22].value := 0.85;
  pos_array[22].label1 := -10;
  for i := 23 to 30 do
    begin
      pos_array[i].value := pos_array[i-1].value - 0.1;
      pos_array[i].label1 := pos_array[i-1].label1 - 10;
    end;
  pos_array[31].value := 0.0;
  pos_array[31].label1 := -100;
end;
{-----}

```

FUNCTION Mean(Values: array_type; Count:integer):Real;

(* This FUNCTION calculates the MEAN of count number of VALUES *)

```

Var
  Accum : Real;                (* sum of all values *)
  I      : Integer;            (* loop control variable *)
Begin (* mean *)
  Accum := 0.0;                (* initialize sum *)
  for I := 1 to Count do
    Accum := Accum + Values[I];
  Mean := Accum / Count;       (* calculate MEAN *)
End; (* mean *)
{-----}

```

FUNCTION Standev(Values: array_type; Mean:REAL; Count:INTEGER):REAL;

(* This FUNCTION calculates the Standard Deviation of COUNT
amount VALUES gives the MEAN *)

```

Var
  Sqr_Accum,                  (* sum of all values squared *)
  temp      : real;
  I          : Integer;       (* loop control variable *)

```

```

Begin (* Standev *)
  Sqr_Accum := 0.0; (* Reset to 0 *)
  For I := 1 To Count Do (* sum up squared values *)
  begin
    temp := (Values[I] - Mean);
    Sqr_Accum := Sqr_Accum + temp * temp;
  end;
  Standev := SQRT( Sqr_Accum / Count) (* Figure Standard Deviation *)
End; (* Standev *)

{-----}
{ This procedure prints a horizontal graph line after all data has been
  totalled
}
procedure get_line( pos_array : grp_array_type;
  var data : data_array_type;
  grp_num,
  data_pts : integer);

var
  1,
  mark_pos : INTEGER;
  line : STRING[80];
  fill : CHAR;
begin
  if pos_array[grp_num].labell = 0 then
    fill := '-'
  else
    fill := ' ';

  for i := 1 to data_pts*5 do
    line[i] := fill;

  write(out_file, pos_array[grp_num].labell:4, ' ');
  for i := 1 to data_pts do
  begin
    if (pos_array[grp_num].value < data[i].value) then
      if not data[i].used then
        begin
          line[i*5 - 1] := mark;
          data[i].used := true;
        end
      else
        line[i*5 - 1] := '.';
    end;
    for i := 1 to data_pts*5 do
      write(out_file, line[i]);
    writeln(out_file);
  end;
}
{-----}

begin
  initial_pos_array(pos_array);
  write('Enter Name of Data File => ');
  readln(file_name);
  assign(in_file, file_name);
  reset(in_file);
  file_name2 := '';

```

```

write('Enter Name for Output File (Ret for .gph ext) => ');
readln(file_name2);
if length(file_name2) = 0 then
    file_name2 := concat(file_name, '.gph');
assign(out_file, file_name2);
rewrite(out_file);
k := 0;
while not eof(in_file) do
begin
    k := k + 1;
    readln(in_file, gamma_0, phi, Me, repeats, runs);
    for I := 1 to runs do
    begin
        readln(in_file, b1[i], b0[i], tsum[i], te[i]);
    end;
    str(repeats, temp_str);
    temp_length := length(temp_str);
    data[k].labell := concat(copy(' ', 1, 3 - temp_length), temp_str);
    data[k].b0_mean := MEAN(b0, runs);
    data[k].b1_mean := MEAN(b1, runs);
    data[k].tsum_mean := MEAN(tsum, runs);
    data[k].te_mean := MEAN(te, runs);
    data[k].b0_std := Standev(b0, data[k].b0_mean, runs);
    data[k].b1_std := Standev(b1, data[k].b1_mean, runs);
    data[k].value := data[k].b0_mean/gamma_0;
    data[k].used := false;
end;
writeln(out_file, '      Graph of % b0 estimate differs from gamma0 : ');
writeln(out_file);
writeln(out_file, ' Gamma_0 = ', gamma_0:3, '  Phi = ', phi:5:3, ' ':10,
    'Source : ', file_name);
writeln(out_file, '      n = ', Me:3, ' ':23,
    'Output : ', file_name2);

writeln(out_file);
writeln(out_file, ' %off');
writeln(out_file);
for i := 1 to num_grps do
begin
    get_line(pos_array, data, i, k);
end;

write(out_file, ' ':5, '-');
for i := 1 to k do
    write(out_file, '-----');
writeln(out_file);

for i := 1 to 3 do
begin
    write(out_file, ' ':7);
    for j := 1 to k do
        write(out_file, ' ', data[j].labell[i], ' ');
    if i = 2 then
        writeln(out_file, ' # repeats')
    else
        writeln(out_file);
end;
end;

```

```

writeln(out_file);

{ print statistical results }
for i := 1 to 94 do
  write(out_file, '-');
writeln(out_file);
writeln(out_file, '
                                b1                                b0'
                                tsum                                te');
writeln(out_file, '      Me      Mean      Std      Mean      ',
              Std      Mean      Mean');
writeln(out_file, '      ---      -----      -----      -----',
              '-----      -----      -----');
for i := 1 to k do
  writeln(out_file, ' :5,data[i].label1, ', data[i].b1_mean:9:8, ',
                  data[i].b1_std:9:8, ', data[i].b0_mean:7:2, ',
                  data[i].b0_std:7:2, ', data[i].tsum_mean:10:1, ',
                  data[i].te_mean:8:1);

close(in_file);
close(out_file);
end.

```

This program generates scatter plots (histograms of the data generated by the BASIC.P program.

```
PROGRAM HISTOGRAM;
```

```
const
  mark = '*';
  num_grps = 20;

type
  table_type      = array[1..num_grps,1..100] of Char;
  count_array_type = array[1..num_grps] of INTEGER;
```

```
var
  gamma_0,
  repeats,
  runs,
  Me,
  group,
  max_grp_cnt,
  i,
  j      : INTEGER;
  phi,
  b1_est,
  b1_max,
  b1_min,
  b0_est : REAL;
  data    : TEXT;
  out_file : TEXT;
  file_name,
  file_name2 : STRING[14];
  count_arr : count_array_type;
  table     : table_type;
  graph_type : CHAR;
  b0_graph  : BOOLEAN;
```

```
{-----}
procedure initialize;
```

```
var
  i, j : INTEGER;
begin
  for i := 1 to num_grps do
    begin
      count_arr[i] := 0;
      for j := 1 to 100 do
        table[i,j] := ' ';
      end;
    end;
```

```
end;
{-----}
```

```
begin
  write('Enter Name of Data File => ');
  readln(file_name);
  assign(data, file_name);
  reset(data);
  write('Enter Name for Output File => ');
  readln(file_name2);
  write('Graph for b0 (Y/N) ? ');
```

```

readln(graph_type);
b0_graph := graph_type in ['Y','y'];
assign(out_file, file_name2);
rewrite(out_file);
while not eof(data) do
begin
  initialize;
  readln(data, gamma_0, phi, Me, repeats, runs);
  for I := 1 to runs do
  begin
    readln(data, b1_est, b0_est);
    if b0_graph then
      group := 1 + trunc( ( (b0_est/gamma_0) * 100.0 - 5.0) / 10)
    else
      group := 1 + trunc( ( (b1_est/phi) * 100.0 - 5.0) / 10);
    if group > num_grps then
      group := num_grps
    else if group < 1 then
      group := 1;
    count_arr[group] := count_arr[group] + 1;
    table[group,count_arr[group]] := mark;
  end;
  max_grp_cnt := 0;
  for I := 1 to num_grps do
    if count_arr[I] > max_grp_cnt then
      max_grp_cnt := count_arr[I];
  if max_grp_cnt < 48 then
    max_grp_cnt := max_grp_cnt + 2;
  if b0_graph then
    writeln(out_file, '          Chart of Estimates of b0 with : ');
  else
    writeln(out_file, '          Chart of Estimates of b1 with : ');
  writeln(out_file);

  writeln(out_file, ' Gamma_0 = ',gamma_0:3, '  Phi = ',phi:5:3, ' ':10,
    'Source : ',file_name);
  writeln(out_file, '          n = ',Me:3, '    r = ',repeats:3, ' ':12,
    'Output File : ',file_name2);
  writeln(out_file);
  for J := 1 to max_grp_cnt do
  begin
    write(out_file, max_grp_cnt - J + 1:2, ' .');
    for I := 1 to num_grps do
      write(out_file, ' ',table[I,max_grp_cnt - J + 1], ' ');
    writeln(out_file, ' .');
  end;
  write(out_file, ' -');
  for I := 1 to num_grps do
    write(out_file, '---');
  writeln(out_file, '---');
  writeln(out_file, ' -90 80 70 60 50 40 30 20 10  0 10 20 30 40 50 60 70 80 90 100+');
  writeln(out_file);
  writeln(out_file, '          % Estimate Differs from Actual Value ');
  writeln(out_file,chr(12));
end;
close(data);
close(out_file);
end.

```

Appendix H - Code/Math Cross-Ref

Equations used in BASIC.P to generate Failure Data

This function is an implementation of equation 7.

$$\beta_0 = \frac{m_e}{1 - \exp(\beta_1 t_e)}$$

Here,

Me - equals the number of faults found,
te - equals the time to end of testing,
bl_est - equals the calculated estimate for bl

```
FUNCTION get_b0_est (Me      : INTEGER;
                    te,
                    bl_est   : REAL) : REAL;

    { function to get estimate of b0 given bl_est }

begin
    get_b0_est := Me / ( 1 - exp(-bl_est * te) );
end;
```

This function is an implementation of equation 11.

$$\frac{m_e}{\beta_1} - \frac{m_e t_e}{\exp(\beta_1 t_e) - 1} - \sum_{i=1}^{m_e} t_i = 0$$

The equation was rearranged to the following form:

$$m_e = \beta_1 \left[\frac{m_e t_e}{\exp(\beta_1 t_e) - 1} + \sum_{i=1}^{m_e} t_i \right]$$

The function has 2 subfunctions, lhs and rhs, which represent each side of the rearranged equation. The bisection method is used to find a point where the values of the lhs and rhs functions are less than a given epsilon.

```
FUNCTION calc_bl_est (Me      : INTEGER;           { # errors found }
                    te,
                    time_sum : REAL) : REAL;      { time to end of testing }
                                                    { sum of failure times }

    { function to get estimate of bl given failure data }

VAR
    bl_est,           { holds working estimate of bl }
    low,              { low end of estimation interval }
    high,             { high end of estimation interval }
    lhs_est,          { var to hold lhs of eq value }
    rhs_est,          { var to hold rhs of eq value }
    Me_x_te           : REAL; { var to hold product calculated once }
```

```

{-----}

FUNCTION lhs (Me          : INTEGER)          : REAL;

    { function to get value of left hand side of equation solving
      for bl }

begin
    lhs := Me;
end; { lhs }

{-----}
FUNCTION rhs (Me_x_te,
             te,
             time_sum,
             bl_est      : REAL) : REAL;

    { function to get value of right hand side of equation solving
      for bl }

begin
    rhs := bl_est * ( Me_x_te / (exp(bl_est * te) - 1) + time_sum);
end;
{-----}
begin
    { initial guess at bl to start calculations }
    bl_est := 0.0001;

    { calculate product once and store for use }
    Me_x_te := Me * te;

    { get left hand side, right hand side guess values }
    lhs_est := lhs (Me);
    rhs_est := rhs (Me_x_te, te, time_sum, bl_est);

    { if lhs_est > rhs_est increase bl_est until not true }
    while lhs_est > rhs_est do
    begin
        bl_est := bl_est + 0.0001;
        lhs_est := lhs (Me);
        rhs_est := rhs (Me_x_te, te, time_sum, bl_est);
    end;
    low := bl_est - 0.0001;
    high := bl_est;

    { perform bisection until value at which relationship first
      changes is found }

    repeat
        bl_est := (low + high) / 2;
        lhs_est := lhs (Me);
        rhs_est := rhs (Me_x_te, te, time_sum, bl_est);
        if lhs_est < rhs_est then
            high := bl_est
        else
            low := bl_est;
        until abs(lhs_est - rhs_est) <= epsilon;

        calc_bl_est := bl_est;
    end;

```

This procedure is an implementation of equation 23. The produces 'repeats' number of failure time data sets and averages the corresponding failure times.

$$t = \frac{-\gamma_0}{\lambda_0} \ln \left(1 + \frac{\ln(1 - r)}{\gamma_0 e^{-(\lambda_0/\gamma_0) t_{sum}}} \right)$$

```

procedure new_data (      gamma_0,                { total number of faults }
                        Me,                { number faults to generate}
                        repeats : INTEGER;    { number of repetitions }
                        var time_sum : REAL;   { sum of all failure times }
                        phi : REAL;          { failure rate per fault }
                        var      t : array_type); { array of generated times }

    {
      procedure to generate a set of failure times using number of repeats
      indicated.
    }

var
  i, j          : INTEGER;    { indices }
  tsum          : REAL;       { value of previous failure time }
  t_avg        : array_type;  { array to hold running total to be averaged }
  bust : boolean;             { indicates overflow condition }

begin
  { initialize array that will hold running totals of times }
  for j := 1 to Me do
    t_avg[j] := 0.0;

  j := 0;

  { get number of time samples indicated by variable repeats }
  repeat
    { initialize variables }
    bust := false;
    tsum := 0.0;
    i := 0;

    { generate required times }
    repeat
      i := i + 1;

      { test for overflow condition }
      if (tsum * phi > 719.0) then
        begin
          bust := true;
        end

      else { generate next time interval }
        begin
          { 1.0 / phi = gamma_0 / lambda_0 }
          t[i] := tsum + (1.0/phi) *
            ln ( 1 - ln(1-random(1.0)) /
              (gamma_0 * exp (-phi * tsum) ) );
          tsum := t[i];
        end;
    until (i = Me) or bust;  { until all times gotten or overflow }

    { add time intervals to t_avg to keep running totals }
    if not bust then
      begin
        for i := 1 to Me do

```

```

        t_avg[i] := t_avg[i] + t[i];
    j := j + 1;
end;

until j = repeats; { repeat until required samples performed }

{ calculate average times and total of time averages }
time_sum := 0.0;
for j := 1 to Me do
begin
    t[j] := t_avg[j] / repeats;
    time_sum := time_sum + t[j];
end;
end;

```

Additional Confidence Interval Program Equations

This function is an implementation of equation 15,

$$I(\beta_1) = m_e \left[\frac{1}{\beta_1^2} - \frac{t_e^2 \exp(\beta_1 t_e)}{[\exp(\beta_1 t_e) - 1]^2} \right]$$

```

FUNCTION I (Me      : INTEGER;
            bl_est,  : REAL) : REAL;
            te      : REAL) : REAL;

    { function to get fisher information for bl }

begin

    I := Me * ( 1 / SQR(bl_est) -
                SQR(te) * exp(bl_est * te) / SQR( exp(bl_est * te) - 1) );
end;
```

The following code segment calculates confidence interval boundaries implementing equation 14.

$$\beta_k \pm \frac{K_{1-\alpha/2}}{\sqrt{I(\beta_k)}}$$

```

I_val := I (num_errors, bl_est[k,j], te[k,j]);
bl_offset := k_val[ii].val / SQRT(I_val);
bl_min := bl_est[k, j] - bl_offset;
bl_max := bl_est[k, j] + bl_offset;
...
b0_min := get_b0_est(num_errors, te[k, j], bl_max);
b0_max := get_b0_est(num_errors, te[k, j], bl_min);
```

End of Document